Solve a Simpler Problem: An Example of Strategy Modeling

One of my favorite problems to model Solve a Simpler Problem with is *Where's Juanita Walking*? It stumps groups of all ages but is accessible to even young students, as the actual calculations only require addition!

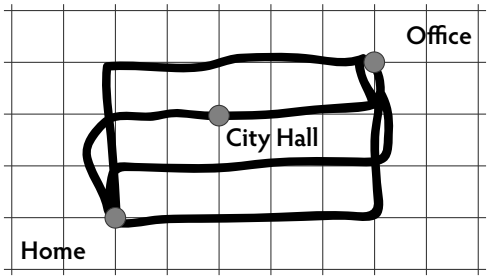
> ***Where's Juanita Walking?***
>
> Juanita's city streets are laid out very neatly on a rectangular grid. Her house is two blocks west and two blocks south of City Hall. Her office is three blocks east and one block north of City Hall.
>
> How many different routes could Juanita take from her house to her office, without going out of her way (meaning without ever going south or west)?

After having students notice and wonder about the problem, I like to have them draw a picture and/or act out the problem, to make sure they have a good visualization of Juanita's city and understand some different possible paths. I often check by asking, "What path would Juanita definitely *not* take?" to make sure the students understand that Juanita does not want to go out of her way. I also want to clear up the idea that Juanita doesn't have to go to City Hall on the way; City Hall is just mentioned to help us understand where her house is in relation to her office.

Most groups try to systematically draw out all the paths they think Juanita could take at this point, or they try to jump directly to a rule; for example, by multiplying the number of blocks east with the number of blocks north that she needs to travel. At this point, I give them very little support so they can really get a feel for what's hard about this problem. However, I will let them know if their answer is incorrect, so that students who've made up rules and don't have a way to test their rules don't feel "over and done" too soon.

Once students have struggled with the problem and are beginning to feel like there's no way they can solve it—an extensive list of paths would be way too exhausting—I then model how I solved the problem. It's a particularly nice problem for me to model because the first time I solved a problem like this was on a final exam. The teacher thought it would be easy but it was really hard! I skipped it and came back to it and spent most of my exam time trying everything I could think of. Here's how it went:

> This problem looks really hard! She could go all the way north first, and then all the way east. That's one way. And all the way east and then all the way north, that's another way. So that's two so far. And then she could go north, then all the way east, then finish the north blocks.
>
> So I could start drawing out the routes she could take:



Ugh, that's just a mess. I'll never keep track of how many routes that way. I could try listing the direction she goes for each block, and make a list that way:

North, North, North, East, East, East, East, East
East, East, East, East, East, North, North, North
North, East, East, East, East, East, North, North
North, North, East, East, East, East, East, North
North, East, North, East, East, East, East, North

I feel like I'm never going to be sure I found them all this way. This problem is really hard!

Wait, I know! When I find a problem that feels too hard, I can use that to my advantage. I can try to look at the problem in a way that makes it simpler.

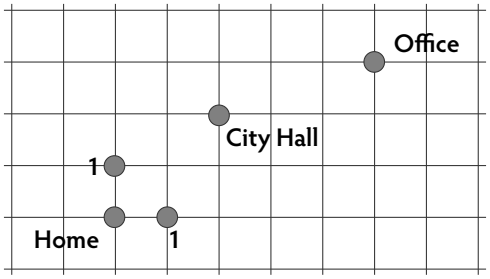
The things that make this problem hard are:

- There are too many routes she could take.
- I won't be sure I found them all.
- It's hard to keep track of the routes I've counted so I don't count them twice.
- Juanita's office is just too far away!

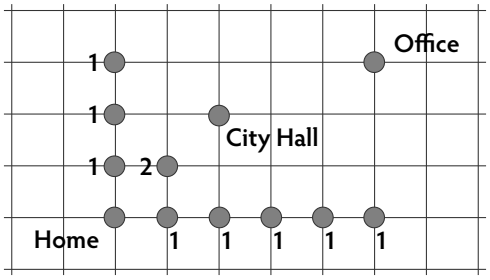
Let me see how I can make each of those things simpler.

This problem is hard because . . .	**I could make it simpler by . . .**	**I'm confident or concerned that's valid because . . .**
There are too many routes to keep track of.	???	
I won't be sure I found them all.	Find a situation where I know I found them all, like a smaller city. Then see if I can apply it to her city.	A little concerned, because what if the pattern doesn't always work?
It's hard to keep track of the routes I've counted so I don't count them twice.	Just using N and E to show which direction she went?	Confident, because they're good abbreviations.
Juanita's office is just too far away!	Move her office closer.	Concerned, because her office really is far away . . . but also confident, because if I get even one location I'm sure of then I can extend that to get to her office.

When I looked at my list, I thought a lot about finding a smaller situation where I am confident I could count all the routes. Even though Juanita's office is five blocks east and three blocks north, I like the idea of pretending it's closer. I can use her same city and just see how many ways there are to get to some easier spots at first.

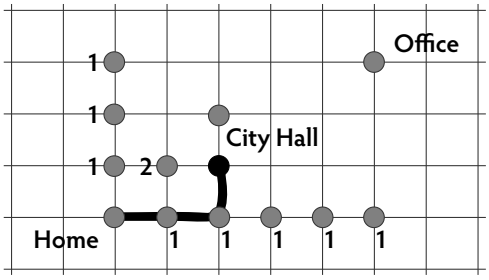


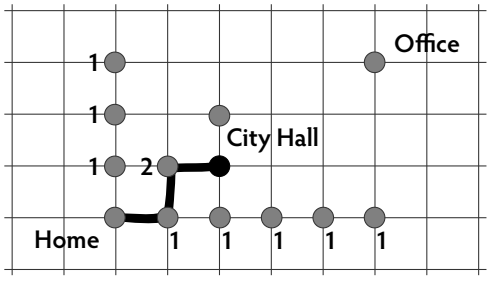
There's definitely only one way to get one block away! Can I add more to that? Like are there any in the inside I could be confident about?

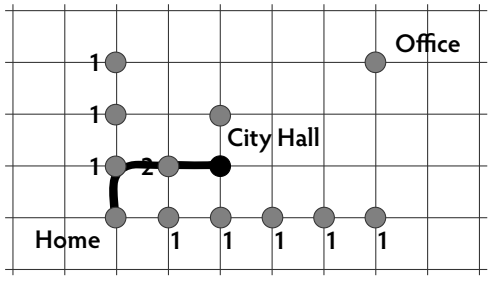


I decided there are two ways to get two blocks away if you go on the diagonal: NE or EN both get you there. But all along the edges there's only one way to get there. NN is the only way to get two blocks north, and EE is the only way to get two blocks east. What about getting two blocks east and one block north? If I can fill in the whole grid this confidently, at least I'll get to Juanita's office eventually!

To get two blocks east and one block north she could go EEN or ENE or NEE. That's three ways. I drew them to be sure:

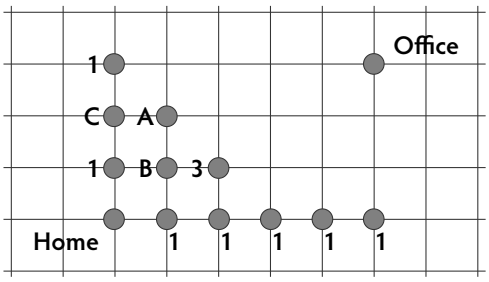






That's when I had an "Aha!" moment. To get to that point, she either had to come from the west or the south. If she came from the south, there was only one way. If she came from the west there were two ways. That's because there are two ways to get to the spot to the west (labeled with 2).

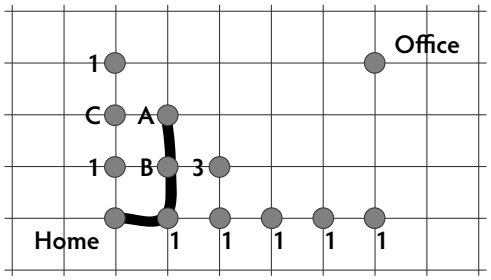
I got rid of City Hall on my map because it was getting in the way, and I started to think about if my pattern really worked.

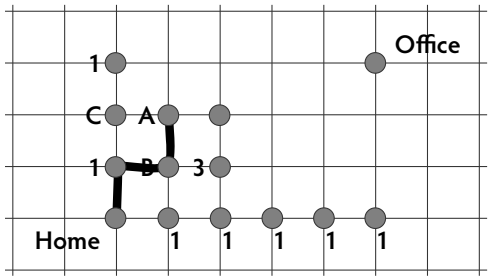


I thought about Point A. I think there are three ways to get there:

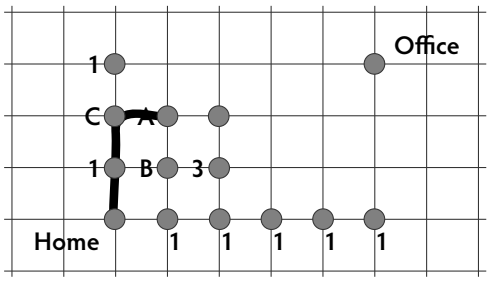
ENN
NEN
NNE

I think there are two ways to get there from the south, because there's two ways to get to Point B, and from Point B to Point A, of course, you have to go north:

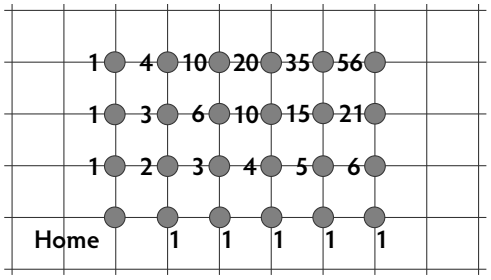




And one way from Point C, because there's one way to Point C from Home and then, of course, you have to go east:



So there's three ways to get to Point A, and that's because there's two ways to get to the point below and one way to get to the point to the left. I can just add the point below and the point to the left and find the number of ways to get to each point. I should be able to finish the grid and find how many ways Juanita can walk to the office!



Using the pattern I found by looking at closer points that I was confident it, I found how many ways Juanita could walk to her office. There are 56 ways. She could walk a different way for 56 days and never run out!

I hope that students notice the three-column chart I used and how I had the idea to try easier problems first, as well as noticing some of the details of how I solved this problem. For follow-up problems, in addition to finding similar "How many paths?" problems, students might be stretched to try Solve a Simpler Problem with problems like: "Six people meet for the first time and shake hands. How many handshakes are there?" or "How many diagonals does a dodecagon (12 sides) have?" or "What is the maximum number of intersection points 10 circles can create?"