Math Fundamentals PoW Packet
Cupcakes, Cupcakes!

Problem 2827   ·   https://www.nctm.org/pows/

Welcome

This packet contains a copy of the problem, the "answer check," our solutions, some teaching suggestions, and samples of the student work we received in January 2003. The text of the problem is included below. A print-friendly version is available using the "Print" link on the problem page.

In *Cupcakes, Cupcakes!*, students are asked to determine whether Andrei will have enough time to finish icing his cupcakes before he has to leave to catch the bus. The **key concepts** are time and rate.

Standards

If your state has adopted the Common Core State Standards, this alignment might be helpful. *Grade 3: Operations & Algebraic Thinking*

   Solve problems involving the four operations, and identify and explain patterns in arithmetic.

Grade 4: Operations & Algebraic Thinking

   Use the four operations with whole numbers to solve problems.

Grade 5: Operations & Algebraic Thinking

   Analyze patterns and relationships.

Mathematical Practices

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
7. Look for and make use of structure.

Cupcakes, Cupcakes!

The Problem

Andrei's school is having a Science Fair this afternoon and he agreed to bring four dozen cupcakes for the reception afterwards. To catch his bus, he needs to be out of the house at 7:50 a.m. He started icing the cupcakes at 7:40 a.m. at a rate of 3 cupcakes per minute.

He soon realized that he wasn't going to finish in time, so at 7:44 his older sister, Zoe, started icing cupcakes too. She iced them at a rate of 4 cupcakes per minute. As they iced the cupcakes, they placed them directly in the boxes that he would carry on the bus. This way, when they iced the last cupcake, Andrei would be ready to dash out the door.

Question: Did they finish in time for Andrei to catch his bus? Explain.

Extra: When did they finish icing the last cupcake?

Answer Check

After students submit their solution, they can choose to "check" their work by looking at the answer that we provide. Along with the answer itself (which never explains how to actually **get** the answer) we provide hints and tips for those whose answer doesn't agree with ours, as well as for those whose answer does. You might use these as prompts in the classroom to help students who are stuck and also to encourage those who are correct to improve their explanation.

   Yes, Andrei would make it out the door with under a minute to spare.

If your answer **doesn't** match ours,

- did you notice that Andrei ices the cupcakes for 4 minutes before Zoe starts helping?

- did you notice that Andrei ices more slowly than Zoe?
- did you try drawing a picture?
- did you use a table to help you organize your work?

If any of those ideas help you, you might *revise* your answer, and then leave a comment that tells us what you did. If you're **still stuck**, leave a *comment* that tells us where you think you need **help**.

If your answer **does** match ours,

- did you try the Extra?
- is your explanation clear and complete?
- did you make any mistakes along the way? If so, how did you find them?
- what hints would you give another student trying to solve this problem?

Revise your work if you have any ideas to add. Otherwise leave us a *comment* that tells us how you think you did—you might answer one or more of the questions above.

Our Solutions

Method 1: Make a Table

We started by making a list of what we noticed when we read the problem:

Andrei said he'd bring four dozen cupcakes to school.
Four dozen is 4 x 12 or 48.
He needs to leave the house at 7:50.
He is icing the cupcakes before leaving.
He started icing at 7:40.
He started icing 10 minutes before he has to leave.
He ices 3 cupcakes in 1 minute.
Zoe helps him ice the cupcakes.
Zoe started icing at 7:44.
Zoe started icing 4 minutes after Andrei started.
She ices 4 cupcakes in 1 minute.
Zoe ices faster than Andrei.

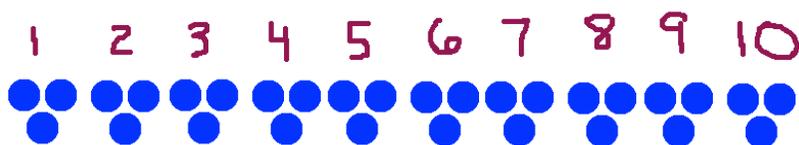We decided to make a table so that we could think about all of these different numbers.

Minutes passed	1	2	3	4	5	6	7	8	9	10
Iced cupcakes by **Andrei**	3	6	9	12	15	18	21	24	27	30
Iced cupcakes by **Zoe**					4	5	12	16	20	24
Total cupcakes iced	3	6	9	12	19	26	33	40	47	54

From our table we can see that at 7:49 Andrei and Zoe will have iced 47 cupcakes and so that leaves just one more cupcake to ice during that last minute Andrei has before leaving the house to catch the bus. Either Andrei could ice it or Zoe could with some seconds to spare. Andrei will be able to catch his bus in time.

Extra: If we assume that Andrei ices that last cupcake, we have to think about how long it takes him to ice one cupcake. There are 60 seconds in a minute and if he ices 3 cupcakes in 1 minute or 60 seconds then he ices 1 cupcake in 20 seconds (since 60 divided by 3 equals 20). The cupcakes would all be iced at 7:49 and 20 seconds or 7:49:20.
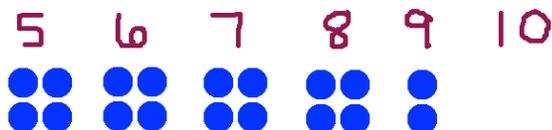
Method 2: Use Manipulatives

After reading the problem it seemed helpful to use counters to think about how this would all work. We got out 48 counters to see how it might work if Andrei iced them by himself. We knew there were 10 minutes of icing time for Andrei and that he could ice 3 cupcakes each minute. We arranged the counters in 3s and had:

We used up 30 counters and had 18 left! We could see why Andrei needed Zoe's help. We knew that there were 6 minutes of icing tie for Zoe and that she could ice 4 cupcakes each minute. We worked

with the 18 counters we had left and had:

$$5 \quad 6 \quad 7 \quad 8 \quad 9 \quad 10$$

We knew that with Zoe's help Andrei would finish in time. In fact, he might even have her do a few more at the end and he would do a few less so that he had a few more seconds to get out the door to catch his bus in time.

Method 3: Logical Reasoning – Separate Times

Andrei has 10 minutes from 7:40 AM to 7:50 AM to get his cupcakes all iced. His icing rate is 3 cupcakes per minute, so he can ice at most 30 cupcakes in 10 minutes (3 x 10 = 30).

When Zoe starts helping there are 6 minutes from 7:44 AM to 7:50 AM. Her icing rate is 4 cupcakes per minute, so she can ice at most 24 cupcakes in 6 minutes (6 x 4 = 24).

When we combine that possible number of cupcakes iced by Andrei and Zoe we have: 30 + 24 = 54. Since 54 is more than 4 dozen (48), Andrei should make it out the door in time to catch his bus.

Method 4: Logical Reasoning – Combined Times

Andrei can ice 12 cupcakes alone from 7:40 between 7:44 because that's 4 minutes at 3 cupcakes per minute and 4 x 3 = 12 cupcakes.

When Zoe joins Andrei they can together ice 7 cupcakes per minute (Andrei can ice 3 and Zoe can ice 4) and they move faster. Between 7: 44 and 7:50, they can together ice 42 cupcakes.

6 minutes x 7 cupcakes/minute = 42 cupcakes

Add the cupcakes Andrei iced alone to the possible number of cupcakes Andrei and Zoe can ice together. A total of 54 (12 + 42) cupcakes could be iced by 7:50

Four dozen equals 48 because 4 x 12 = 48. Since 54 is greater than 48, Andrei will finish icing cupcakes in time to leave the house and catch the bus.

Teaching Suggestions

When we first offered this problem many students approached the problem from the perspective of icing for the entire time given. How many cupcakes could be iced if they worked right up to 7:50? There are a few ways to this figure out including calculating Andrei's cupcakes separately from Zoe's or calculating Andrei's total when he iced alone and then combining to get a total when the siblings iced together.

A "minute by minute" approach was also popular. Many submitters constructed a chart of some type for this method. Some charts showed the number of cupcakes that are iced *during* each minute. Some charts showed the number that were iced *by* a particular minute. As you might imagine students who were consistent with *how* they entered the numbers into their charts, were more successful than those who didn't pay attention to that detail. The concept of time on the clock and duration of elapsed time confused a few problem solvers.

One interesting twist came from some submitters who had focused on the combined rate of 7 cupcakes per minute. When they thought about time needed for that last cupcake, they simply took 1/7 of a minute and converted that to the rounded value of 9 seconds if both iced it together at their prescribed rates. While mathematically this might be correct, it presents a good opportunity to think about how we use mathematics to model real situations. It would, in fact, be difficult to have two siblings work together on one small cupcake while maintaining their efficiency and constant rates of icing!

Another twist came from some students who suggested that Andrei stop after doing 24 cupcakes. They understood that Zoe could do 24 (6 minutes • 4 cupcakes) on her own, so that Andrei would have enough if he worked until he had also completed 24 (8 minutes • 3 cupcakes). This makes for a fair division of labor.

In the solutions below, I've provided the scores the students would have received in the **Interpretation** category of our scoring rubric. My comments focus on what I feel is the area in which they need the most improvement.

Novice	Apprentice	Practitioner	Expert
Understands few of the criteria listed in the Practitioner column.	Understands most but not all of the criteria listed in the Practitioner column.	Understands that • the goal is to explain if Andrei will finish in time to catch his bus. • four dozen cupcakes is 48 cupcakes. • Andrei ices 3 cupcakes a minute. • Zoe ices 4 cupcakes a minute. • Andrei will ice alone 4 minutes, then Zoe and Andrei will ice together for the remaining time.	Is at least a Practitioner in Strategy and comes up with the correct solution for the Extra.

Bobby

age 12

Interpretation
Novice

Zoe and andries didn't make because there one cupcake short.

3*9=27+4*5=20=47 there one short

I notice that Bobby may be using the 3 cupcakes per minute and 4 cupcakes per minute rates for Andrei and Zoe but he's run his numbers and symbols together into one string.

I would first draw his attention to this string of numbers and symbols. I'd ask him to tell me more about what information he used from the problem. I'd suggest that he separate each operation on a separate line as he tells me more about it.

Kyla

age 10

Interpretation
Novice

Yes, the did finish in time

To get my answer I drew 4 dozen cupcakes and circled them as he iced them to find out if he did it in time to catch the bus.

Kyla has a great idea about drawing a picture to help think about this problem. Because she's said so little, however, I'm not sure if she knows how many cupcakes are in 4 dozen. I wonder if she included Zoe in the icing task since she only mentions "he" who I assume is Andrei.

I would encourage her to tell me more about her drawing. What did she circle and how did she think about it?

Andy age 11 Interpretation **Apprentice**	Yes he did have enough time to finish The first thing I did was 40 to 41is3 cup cakes. Then 41to 42 minutes is 6 cup cakes.Then 42 to 43 minutes is 9 cup cakes.Then 43 to 44 minutes is 12 cup cakes. Then 44 to 45 to minutes is 19 cup cakes.Then 45 to 46 minutes is 26 cup cakes.Then 46 to 47 minutes is 33 cup cakes. Then 47 to 48 is 40 cup cakes.Then 48 to 49 minutes is 47 cup cakes Then 49 to 50 minutes is 54 cup cakes. So he had 6 minutes left.	*I notice that Andy is thinking only in terms of the minutes after the hour and doesn't include any mention of the hour. Once I got used to that it was easier to follow his thinking.* *Once he gets to the beginning of the 5th minute I notice he switches from adding 3 more cupcakes to adding 7 more cupcakes.* *Those are more Clarity or Completeness issues but his final sentence makes me wonder if he lost his train of thought of what he's counting. It's true that 54 is 6 more than 48 but I wonder how that changed to minutes in his mind!*
Kevin age 10 Interpretation **Apprentice**	No because they would have to do 48 cupcakes in 10 minutes. They would do 7 in 2 minutes. They would do 35 cupcakes and need 13 more. First I did 12 x 4 and got 48. Next I did 4 + 3 and got 7. I got 7 because it said his sister came and helped and did 4 a minute and he 3 a minute. Then i did 7 x 5 and got 35. I did that because they did 7 in 2 minutes and 5 times 2 and got 10 minutes. My answer is no.	*I notice that Kevin understands how many cupcakes are in a 4 dozen. He also understands that when Zoe and Andrei work together they can ice 7 cupcakes in a minute.* *I wonder if he realizes that Andrei is going to ice alone the first four minutes and then he and Zoe are going to ice together for the remaining 6 minutes.* *I might suggest that he draw a picture, use manipulatives, or draw a timeline.*
Rebecca age 12 Interpretation **Apprentice**	No, they did not make it in time for Andrei to finish the batch of cupcakes. And they ended at 7:52. First, you have to think how many 4 dozen is, it comes out to be 48. Then, you have to figure out how many cupcakes Andrei can make in 10 minutes which is 30. Then, since his sister started at 7:44 and could make 4 cupcakes in 1 minute, you knew that she make 24. Then you add thoes together and you get 54 which is 14 cupcakes short. Then, you had to figure out how Andrei and Zoe split 14 in half so i figured that Andrei would make 6 more cupcakes which would take 2 mintues and Zoe would make 7 cupcakes in 2 minutes but there would be a few extra seconds left so Andrei and Zow were finished icing cupcakes at about 7:52.	*Rebecca has a strong start explaining how she's using the information given in the problem.* *I confess that she lost me right after her claim that 54 is 14 cupcakes short.* *I think she's farther along in her thinking than Kevin but using a manipulative or making a drawing or timeline could also help her think more about the problem.*

Brian

age 10

Interpretation

Practitioner

Andrei and Zoe finished icing the cupcakes, and he made the bus.

They finished because Andrei made three cups a minute for four minutes. So you would multiply three cupcakes a minute times four minutes and you will get twelve cupcakes. Then Zoe came and she makes four cupcakes a minute, and Andrei makes three cupcakes a minute. So, you would add three plus four which equals seven, so you know how much they make together in a minute. Since there is six minutes left you multiply seven cupcakes a minute times six minutes which equals forty-two. Finally you add the amount of cupcakes Andrei made by himself plus the amount he and Zoe made together which is fifty-four because 12+42=54. If four dozen is forty-eight and they made fifty-four, and fifty-four is greater than forty-eight, than that means that they finished in time.

Bailey

age 10

Interpretation

Practitioner

My answer is yes they did ice all 4 dozen cupcakes in time to get on the bus.

The question is: Did they finish in time for Andrei to catch his bus? The problem is :How many cupcakes did both of them get iced in each minute?

The answer is: They did achieve thier goal of icing all 48 cupcakes in the 10 minutes.
I know:
- They had 10 minutes to ice four dozen cupcakes
- They have to ice 48 cupcakes

MY EXPLINATION AND MY VERIFICATION:

1. First, I need to know how many cupckes he iced in the first 4 minutes. We only need to know how many he did because his sister, Zoe, did not start to ice untill 7:44. If he did 3 cupcakes in one minute, then you would have to multiply that by the number of minutes he had before his sister came.

3 cupcakes x 4 minutes=12 cupcakes

2. Second, I need to know how many cupcakes got iced when his sister was helping him for the last 6 minutes he had before school. She was able to ice 4 cupcakes per minute. Since she did 4 cupcakes in one minute, I need to multiply the number of cupcakes per minute by the number of minutes that she had left . (she had 6 minute left)

4 cupcakes x 6 minutes=24 cupcakes

3. Third, I need to know how many cupcakes Andrei did in the last 6 minutes as his sister was helping him. If he did 3 cupcakes a minute in 6 minutes, I need to multiply 3 cupcakes by 6 minutes.

3 cupcakes x 6 minutes=18 cupcakes

4. I need to add all of the answers together to see if they iced all of the cupcakes before ten minutes.
 12+24+18=54 cupcakes
Since the answer I got was over the four dozen, then I think they iced all 48 cupcakes in or less than ten minutes.

Ryan
age 11

Interpretation
Practitioner

Yes he did finish in time to catch his bus.

I did this by knowing he can ice 3 cupcakes a minute. So it was 7:44 when his sister started to help him. So I multiplied 4 minutes x 3 and my product was 12. At 7:44 his sister started to help and she can ice 4 cupcakes per minute, if Andrei can ice 3 cupcakes per min. and his sister can ice 4 cupcakes per min. then that equals seven. So they ice 7 cupcakes per min. if Andrei already iced 12 then

7+12=19 and thats 7:45.
19+7=26 and thats 7:46.
26+7=33 and thats 7:47.
33+7=40 and thats 7:48.
40+7=47 and thats 7:49.

Then theres 1 more cupcake left which takes him less than a minute, and he is out the door by 7:50.

Ryan's explanation includes a nicely formatted list that helps the reader know what could be happening each minute that Andrei and Zoe are icing together.

One small detail for Ryan to raise his Completeness score is to include how he knew the target number of cupcakes is one more than 47. It's a small detail to mention the 4 dozen idea and yet important.

Giri
age 8

Interpretation
Expert

Yes, Andrei finishes all cupcakes in time to catch the bus.

Andrei has to ice 4 dozen = 48 cupcakes.
At 7:40AM , he starts icing the cupcakes @ 3 cupcakes/minute.
At 7:44AM , he completes 4 * 3 = 12 cupcakes.
       His sister Zoe comes to help him.
       She completes @ 4 cupcakes/min.
       Together , they complete (4+3=7) 7 cupcakes/minute.
They still have 6 minutes left.
In 6 minutes , they can complete (7*6=42) 42 cupcakes.
Since Andrei already finishes 12 cupcakes before Zoe joins him,
so at 7:50AM , they would have completed
       (12 + 42 = 54) 54 cupcakes.
But they only have to ice 48 and since they could ice 54 cupcakes
and they will be able to finish in time for Andrei to catch the bus.

Extra:
======
They would ice the last cupcake between 7:49:15 AM
Explanation:
==========
At 7:40AM   Andrei starts icing
At 7:44AM   He completes 12 cupcakes,Zoe joins him
At 7:45AM   12 + 7 = 19 cupcakes are completed
At 7:46AM   19 + 7 = 26 cupcakes are completed
At 7:47AM   26 + 7 = 33 cupcakes are completed
At 7:48AM   33 + 7 = 40 cupcakes are completed
At 7:49AM   40 + 7 = 47 cupcakes are completed
At 7:50AM   47 + 1 = 48 cupcakes are completed

Assuming Zoe completes the last cupcake, because she takes less time.
Andrei finishes 1 cupcake in 20 (60/3 = 20) seconds.
Zoe   finishes 1 cupcake in 15 (60/4 = 15) seconds.
So Zoe is less. So they complete the icing job at 7:49:15AM

Giri has done a great job with the main part of the problem and also the Extra.

Scoring Rubric

A **problem-specific rubric** can be found linked from the problem to help in assessing student solutions. We consider each category separately when evaluating the students' work, thereby providing more focused information regarding the strengths and weaknesses in the work.

We hope these packets are useful in helping you make the most of Math Fundamentals Problems of the Week. Please let me know if you have ideas for making them more useful.

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