Pre-Algebra PoW Packet

Trick-or-Treat Route

Problem 410 • https://www.nctm.org/pows/

Welcome

This packet contains a copy of the problem, the "answer check," our solutions, some teaching suggestions, and samples of the student work we received in December 1999. The text of the problem is included below. A print-friendly version is available using the "Print" link from the blue-shaded box on the problem page.

Standards

In ***Trick-or-Treat Route***, students are given a map of possible trick-or-treat routes with times between the different houses and asked how many different possible routes there are. Also, they're asked to note the shortest route and the longest route. The **key concept** is graph theory.

If your state has adopted the Common Core State Standards, this alignment might be helpful:

Mathematical Practices

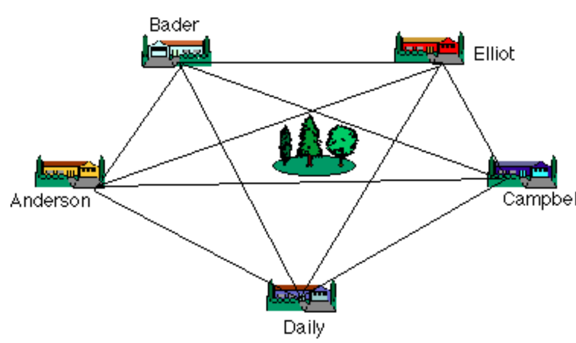
1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics

The Problem

Trick-or-Treat Route

Mrs. Anderson told her son Todd that he was going to have to take his little sister, Grace, trick-or-treating around their neighborhood on Halloween night. His mother agreed that he only had to take her to four houses, and then he could go out with his friends.

Todd wants to plan ahead, so today his friend Joe helped him create a map of the neighborhood with all the houses he must visit with Grace.



Next they made a list of the time it took them to walk between each two houses.

- Anderson to Bader ---- 5 minutes
- Anderson to Campbell ---- 9 minutes
- Anderson to Daily ---- 6 minutes
- Anderson to Elliot ---- 10 minutes
- Bader to Campbell ---- 12 minutes
- Bader to Daily ---- 11 minutes
- Bader to Elliot ---- 5 minute
- Campbell to Daily ---- 6 minutes
- Campbell to Elliot ---- 4 minutes
- Daily to Elliot ---- 8 minutes

When Grace saw what Todd was doing, she decided to plan her own route to maximize the time she will be out trick-or-treating.

Find the answers to each of the following questions. Remember that in each case they must start and end at the Anderson house.

- How many different routes could the children take?
- What is the shortest route based on the times given?
- What is the longest route based on the times given?

Answer Check

After students submit their solution, they can choose to "check" their work by looking at the answer that we provide. Along with the answer itself (which never explains how to actually **get** the answer) we provide hints and tips for those whose answer doesn't agree with ours, as well as for those whose answer does. You might use these as prompts in the classroom to help students who are stuck and also to encourage those who are correct to improve their explanation.

> There are 24 routes are possible. The shortest takes 26 minutes and the longest takes 50 minutes.

If your answer **doesn't** match ours,

- did you decide that going in one direction or the reverse direction would be the same? If you've explained that assumption, 12 routes is another possible answer.
- did you notice the children always started and ended at the Anderson house?
- did you make a list to keep track of your thinking?

If any of those ideas help you, you might *revise* your answer, and then leave a comment that tells us what you did. If you're **still stuck**, leave a *comment* that tells us where you think you need help.

If your answer **does** match ours,

- are you confident that you could solve another problem like this successfully?
- is your explanation clear and complete?
- did you make any mistakes along the way? If so, how did you find them?
- what hints would you give another student trying to solve this problem?

Revise your work if you have any ideas to add. Otherwise leave us a *comment* that tells us how you think you did—you might answer one or more of the questions above.

Our Solutions

Method 1: Make an Exhaustive List

We made a list of all the different routes using these abbreviations: Anderson (A), Bader (B), Campbell (C), Daily (D), and Elliot (E). In each case we knew that we had to start at A. Starting at A there are four choices for the first leg of the route. Once this way is chosen there are three choices for the second leg of the route, then two choices for the third leg of the route and one choice for the fourth leg of the route.

These are the routes and the time it takes for each round trip:

A-B-C-D-E-A	5+12+6+8+10=41
A-B-D-C-E-A	5+11+6+4+10=36
A-B-E-D-C-A	5+5+8+6+9=33
A-B-E-C-D-A	5+5+4+6+6=26
A-B-D-E-C-A	5+11+8+4+9=37
A-B-C-E-D-A	5+12+4+8+6=35
A-C-B-D-E-A	9+12+11+8+10=50
A-C-B-E-D-A	9+12+5+8+6=40
A-C-E-D-B-A	9+4+8+11+5=37
A-C-E-B-D-A	9+4+5+11+6=35
A-C-D-E-B-A	9+6+8+5+5+=33
A-C-D-B-E-A	9+6+11+5+10=41
A-D-B-C-E-A	6+11+12+4+10=43
A-D-C-B-E-A	6+6+12+5+10=39
A-D-B-E-C-A	6+11+5+4+9=35
A-D-C-E-B-A	6+6+4+5+5=26

A-D-E-B-C-A 6+8+5+12+9=40
A-D-E-C-B-A 6+8+4+12+5=35
A-E-B-C-D-A 10+5+12+6+6=39
A-E-B-D-C-A 10+5+11+6+9=41
A-E-C-B-D-A 10+4+12+11+6=43
A-E-C-D-B-A 10+4+6+11+5=36
A-E-D-B-C-A 10+8+11+12+9=50
A-E-D-C-B-A 10+8+6+12+5=41

The children could take 24 different routes. The shortest route is 26 minutes long (Anderson-Bader-Elliot-Campbell-Daily-Anderson), and the longest route is 50 minutes long (Anderson-Elliot-Daily-Bader-Campbell-Anderson).

Method 2: Make a List but Look for a Pattern

After reading the problem we began by making a list. If you start with a (Anderson) and then go to b (Bader), there are six different ways to go with those two as the first and second:

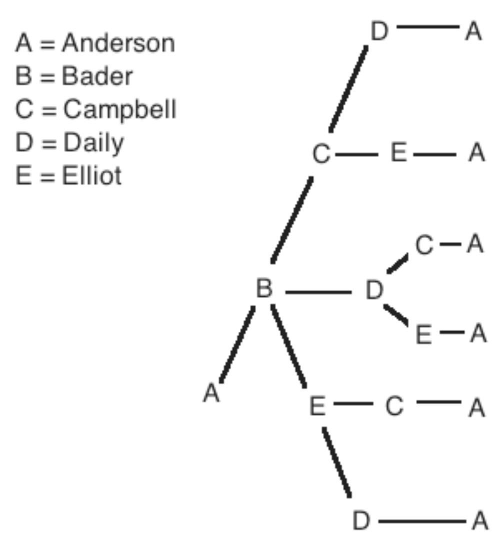
a-b-c-d-e-a
a-b-c-e-d-a
a-b-d-c-e-a
a-b-d-e-c-a
a-b-e-c-d-a
a-b-e-d-c-a

We saw a pattern as we thought about doing this for each of the houses. We thought if c (Campbell) were listed second instead of b, we would again generate 6 choices. Similarly, if we started with d (Daily) or e (Elliot) we would generate 6 choices for each. We knew there are 6 + 6 + 6 + 6 choices for a total of 24.

We found the shortest route by starting out from Anderson's and always picking the shortest "next leg" possible until we went to all houses and got back to Anderson's. It was 26 minutes. We got the longest route by doing the same thing, just picking the longest "leg" each time. It was 50 minutes.

Method 2: Make a Tree Diagram

The way that we worked this out is that we drew a tree with Anderson as the main base and branching to Bader, Campbell, Daily and Elliot. Off the Bader's house we had Campbell, Daily, Elliot and back home to the Anderson's. It looked like this:



We did this starting with A each time but going first to C and then instead going first to D and the last tree showed going first to E. In each case there were 6 paths. The total number of paths in all was 24 that the children could take.

To work out each house, and back to the shortest route i just went along the shortest path for start at Anderson to Bader to Elliot to Campbell to Daily Anderson, 5 + 5 + 4 + 6 + 6 = 26 minutes

To work out each house, and back to the longest route i just went along the longest path for start at Anderson to Elliot to Daily to Bader to Campbell Anderson, 10 + 8 + 11 + 12 + 9 = 50 minutes

Method 3: Use the Multiplication Principle

The children are at the first house (Anderson). There are 4 choices for the second stop. I wrote down four. From one of those houses, there are three choices for the third stop. I wrote down three. From one of those houses, there are two choices for the fourth stop. I wrote down two. From one of the two houses, there is only one stop left. I wrote down one. I know that when you can list the choices like that you can find the total number of combinations by multiplying the numbers. So, I multiplied all of them. 4 x 3 x 2 x 1 = 24. I got 24 possible routes.

I figured out that the shortest route was from Anderson to Bader to Elliot to Campbell to Daily and back to Anderson because they used the smallest times from one house to the other.

To figure out the shortest and longest routes I wrote all the times for houses in order: (4, 5, 5, 6, 6,) 8, 9, 10, 11, 12. The smallest times are between (and) and the sum of the minutes is 26.

I figured out that the longest route was from Anderson to Elliot to Daily to Bader to Campbell and back to Anderson because they used the largest times from one house to the other. All the times for houses: 4, 5, 5, 6, 6, <8, 9, 10, 11, 12> The largest times are between < and > and the sum is 50 minutes.

I noticed when I looked at the diagram that the smallest times made a pentagon and the largest times made a star.

Teaching Suggestions

When we first offered this problem some students considered “different” to mean that going in one direction, for example, A-B-C-D-E-A is different from A-E-D-C-B-A. Other students considered a route and its reverse to count as just one possibility and so their answers were ½ as large as the others. As long as the students explained their process, we accepted either answer.

Common mistakes in problem solving included not having the children return to the Anderson house, not answering all three questions, and getting mixed up on the number of possible routes because they didn’t have a systematic approach to keeping a list and that was the strategy they chose. Common mistakes in communication included not including enough explanation so that another student would be able to know how the problem had been solved.

The questions in the Answer Check, above, might serve as good prompts to help students make progress. Encourage students to use a strategy that works for them. You can see from the various methods that we have thought to use for this problem that there are several ways to approach this problem. And keep in mind that we may not have thought of them all!

Sample Student Solutions focus on Clarity

In the solutions below, I’ve provided the scores the students would have received in the **Clarity** category of our scoring rubric. My comments focus on what I feel is the area in which they need the most improvement.

Novice	Apprentice	Practitioner	Expert
Explanation is very difficult to read and follow.	Another student might have trouble following the explanation. Long and written in one paragraph. Many spelling errors/typos.	Explains the steps that they *do* explain in such a way that another student would understand (needn’t be complete to be clear). Makes an effort to check formatting, spelling, and typing (a few errors are okay).	Format and organization make ideas exceptionally clear. Answer is very readable and appealing.

Brad
age 12

Clarity
Novice

I got 24 routes. I got this using a tree strategy.

I put a big A then i just branched off of that until i got the last a's and then counted them up.

Using a tree diagram is a great strategy for this problem. I know that it can be challenging to include a diagram but Brad might be encouraged to take a photo of it and upload the graphic. Without some sample of the diagram or description of it, another student might have no idea about how that strategy can help you find the answer.

Henry
age 12

Clarity
Novice

There are 5 different routes they could take. The shortest route will take 14 minutes. The longest route will take 31 minutes.

For #1, they could go to every house except Daily, every house except Bader, every house except Anderson, every house except Campbell, and every house except Elliot. That's 5 houses.For #2, They could go to Campbell, then to Elliot, then to Bader, then to Anderson. That's 14 minutes.

For #3, They could go to Campbell, then to Bader, then to Daily, then to Elliot. That's 31 minutes.

I notice that Henry mentions "every house except" but I'm not sure why that is how he approached the problem. I'm also unclear on how counting those 5 houses changes into 5 routes. Houses and routes are different units.

Alex
age 11

Clarity
Apprentice

1:there are 120 possible routes for the chidren to take. 2:the shortest route is A-C-B-D-A that takes 26 minutes. 3:the longest route is A-C-B-D-E-A that takes 5o minutes.

For number 1, from anderson there are 5 routes * 4 from the next house * 3 for the next house *2 for the next *1 for the last. Its permutations.for #2, around the edges is the shortest route.for #3 going through the town was the longest route.

I notice that Alex states that the problem has to do with permutations. I wonder if a classmate would understand what he meant by that word. I think it might help him to understand the problem more if he tried listing some of the routes. I wonder if he would realize that 120 possible routes is too large!

Katie
age 13

Clarity
Apprentice

There are 24 possible routes the children could take. They shortest route is the route around the perimeter. The longest route is the route from Anderson to Elliot to Daily to Bader to Campbell back to Anderson.

I found out that there are 24 possible routes the children could take by drawing out all the possible number of routes on a peice of paper. I got 12 routes and the I noticed that you could also go the reverse way. So I multiplied 12*12 and got a total of 24 routes.

I found out the shortest route and the longest route by adding up all the amounts of time that each route would take. The shortest route is the route around the perimeter. It is from Anderson to Bader to Elliot to Campbell to Daily back to Anderson. It would take 26 minutes. The longest possible route is the star route. From Anderson to Elliot to Daily to Bader to Campbell back to Anderson which took 50 minutes.

I notice that Katie multiplied 12•12 and got 24 and yet when I multiply those two numbers I get 144. I might either ask her why she multiplied or how 12 "times" 12 can equal 24.

Besides that one mis-step affecting her Clarity score, I would encourage her to also work on her Completeness score. Could she talk more about how she drew the routes? Or, better yet, might she include a photo of her drawing?

TJ
age 12

Clarity
Apprentice

They could take 24 different routes. The shortest route based on the times given is 26 min.to go all the way around the pentagon. The longest route is 50 min. to go all the way around the star.

To find out how many different routes there were I just for got about the andersons, because they would be in all the routes any way. I looked to see how many different way there were from the Baders and there were 6. I just figured that that was the same amount for all of the other places and timesed 6 by 4 and got 24. How I found the answer was I looked at the shorter amounts of time to get to house to house and contected the shorter times, and I did the same thing with the longer time.

Like Katie, TJ could work on making his explanation just a little more complete. Including the details of the different routes if the Baders were chosen as the second house could help another student see that there were 6.

To improve his Clarity score, TJ should carefully check his spelling. I would also ask him to find an alternative for the word "timesed."

Ilan
age 12

Clarity
Practitioner

There are 12 different routes that the children could take. The shortest route is from Anderson to Bader to Elliot to Campbell to Daily to Anderson, or the other direction. The longest route is from Anderson to Elliot to Daily to Bader to Campbell to Anderson, or the other direction.

First, I noticed that, like last week's problem, this problem involved permutations. I changed Anderson, Baily, Campbell, Daily, and Elliot to A, B, C, D, and E. I arranged my set-up like this:
1. A-B-C-D-E-A 2. A-B-C-E-D-A 3. A-B-D-C-E-A 4. A-B-D-E-C-A
5. A-B-E-C-D-A 6. A-B-E-D-C-A

In the set-up starting with A-B, there were 6 possibilties. Therefore, in the set-up starting with A-C, there would also be 6, and same with A-D and A-E. I tested this and it worked. 6x4=24, so there are 24 permutations. However, for each permutation that I found, there was another one that was the same, only backwards. Therefore I divided 24 by 2 and got 12.

Based on my permutations, I could draw each route. From there I added up the amount of minutes each route took and came out with Anderson to Bader to Elliot to Campbell to Daily to Anderson being 26 minutes, and Anderson to Elliot to Daily to Bader to Campbell to Anderson being 50 minutes. Also I noticed that the shortest route involved only the outsides (the shortest distances), and that the longest route involved only the insides (the longest distances).

Ilan's solution is an example of assuming that a "backwards" route shouldn't be counted.

He's done a solid job clearly explaining his thinking.

John
age 13

Clarity
Expert

There are 24 different routes the children can take.
The shortest route takes 26 minutes. There were two different routes that took 26 minutes. The first was from the Anderson's to the Bader's to the Elliot's to the Campbell's to the Daily's back to the Anderson's. The second was from the Anderson's to Daily's to the Campbell's to the Elliot's to the Bader's back to the Anderson's.

The longest route takes 50 minutes. There are two routes. The first is from the Anderson's to the Campbell's to the Bader's to the Daily's to the Elliot's back to the Anderson's. The second is from the Anderson's to the Elliot's to the Daily's to the Bader's to the Campbell's and back to the Anderson's. Although there are two routes they are doing the same houses in reverse.

To get the above answers I made an organized list and placed the times

John's format and organization make his solution exceptionally clear.

given to travel between each house beneath and then I added them together. Because each family's last name began with a different letter of the alphabet I used for the Anderson's A; Bader's B; Campbell's C; Daily's D; Elliot's E.

I also found that to go from the Anderson to a set second house always gave six ways. Therefore 6 times 4 different houses = 24

Anderson to Bader:

A to B to C to D to E to A
5 + 12 + 6 + 8 + 10 = 41
A to B to C to E to D to A
5 + 12 + 4 + 8 + 6 = 35
A to B to D to C to E to A
5 + 11 + 6 + 4 + 10 = 36
A to B to D to E to C to A
5 + 11 + 8 + 4 + 9 = 37
A to B to E to D to C to A
5 + 5 + 8 + 6 + 9 = 33
A to B to E to C to D to A
5 + 5 + 4 + 6 + 6 = 26*

Anderson to Campbell
A to C to B to D to E to A
9 + 12 + 11 + 8 + 10 = 50*
A to C to B to E to D to A
9 + 12 + 5 + 8 + 6 = 40
A to C to D to B to E to A
9 + 6 + 11 + 5 + 10 = 41
A to C to D to E to B to A
9 + 6 + 8 + 5 + 5 = 33
A to C to E to D to B to A
9 + 4 + 8 + 11 + 5 = 37
A to C to E to B to D to A
9 + 4 + 5 + 11 + 6 = 35

Anderson to Daily
A to D to C to B to E to A
6 + 6 + 12 + 5 + 10 = 39
A to D to C to E to B to A
6 + 6 + 4 + 5 + 5 = 26*
A to D to B to C to E to A
6 + 11 + 12 + 4 + 10 = 43
A to D to B to E to C to A
6 + 11 + 5 + 4 + 9 = 35
A to D to E to B to C to A
6 + 8 + 5 + 12 + 9 = 40
A to D to E to C to B to A
6 + 8 + 4 + 12 + 5 = 35

Anderson to Elliot

A to E to C to D to B to A

10 + 4 + 6 + 11 + 5 = 36

A to E to C to B to D to A

10 + 4 + 12 + 11 + 6 = 43

A to E to D to C to B to A

10 + 8 + 6 + 12 + 5 = 41

A to E to D to B to C to A

10 + 8 + 11 + 12 + 9 = 50*

A to E to B to C to D to A

10 + 5 + 12 + 6 + 6 = 39

A to E to B to D to C to A

10 + 5 + 11 + 6 + 9 = 41

The shortest route takes 26 minutes.

The longest route takes 50 minutes

Scoring Rubric

A **problem-specific rubric** can be found linked from the problem to help in assessing student solutions. We consider each category separately when evaluating the students' work, thereby providing more focused information regarding the strengths and weaknesses in the work.

We hope these packets are useful in helping you make the most of Pre-Algebra Problems of the Week. Please let me know if you have ideas for making them more useful.

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