

## Problem of the Week Teacher Packet

# Eating Grapes

On Monday Angela ate some grapes. On Tuesday she was hungrier and ate six more grapes than she ate on Monday. Each day that week she ate six more grapes than the day before. After she had eaten her grapes on Friday she had eaten 100 grapes in all.

How many grapes did she eat on Monday?

**Extra:** If she continues this pattern, on each day eating six more grapes than the day before, on which day would she eat her 300th grape?



### Answer Check

After students submit their solution, they can choose to “check” their work by looking at the answer that we provide. Along with the answer itself (which never explains how to actually get the answer) we provide hints and tips for those whose answer doesn’t agree with ours, as well as for those whose answer does. You might use these as prompts in the classroom to help students who are stuck and also to encourage those who are correct to improve their explanation.

Angela ate 8 grapes on Monday.

If your answer **doesn’t** match our answer,

- did you remember that the number of grapes eaten each day after Monday is six more than was eaten the day before?
- did you understand that the total of the grapes eaten over five days is 100?
- did you check your arithmetic?

If any of those ideas help you, you might revise your answer, and then leave a comment that tells us what you did. If you’re still stuck, leave a comment that tells us where you think you need help.

If your answer **does** match ours,

- have you clearly shown and explained the work you did?
- did you make any mistakes along the way? If so, how did you find and fix them?
- are there any hints that you would give another student?
- does this problem remind you of experiences you’ve had?
- did you try the Extra?

Revise your work if you have any ideas to add. Otherwise leave us a comment that tells us how you think you did—you might answer one or more of the questions above.

### Our Solutions

#### Method 1: Notice and Wonder®

Our group used the Activity Series worksheet that our teacher gave us and we tried to notice and wonder everything that we possibly could. Here is our list of noticings:

- Angela ate grapes.
- She ate grapes on Monday.
- On Tuesday she ate 6 more grapes than what she ate Monday.

- On Wednesday she ate 6 more grapes than Tuesday.
- On Thursday she ate 6 more grapes than Wednesday.
- On Friday she ate 6 more grapes than Thursday.
- She ate 100 grapes total.

We decided that we could put our information all in a table to think about it more.

Days:	Monday	Tuesday	Wednesday	Thursday	Friday
Grapes Eaten:	?	? + 6	? + 6 + 6	? + 6 + 6 + 6	? + 6 + 6 + 6 + 6

If we add the grapes she ate each day, it should equal the total of 100.

$$? + ? + 6 + ? + 6 + 6 + ? + 6 + 6 + 6 + ? + 6 + 6 + 6 + 6 = 100$$

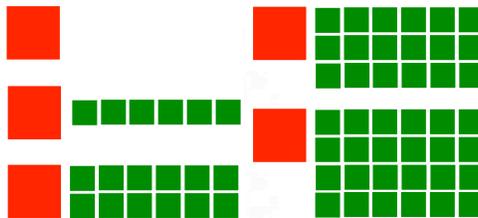
We counted five ? and ten 6s.

$$? + ? + ? + ? + ? + 60 = 100$$

We noticed that the five ? would have to equal 40. Since five 8s make 40 we decided Angela ate 8 grapes on Monday.

### Method 2: Using Manipulatives

We used a red block to represent the number of grapes Angela ate on Monday. For Tuesday we used a red block and six green blocks to show she ate six more grapes than on Monday. For Wednesday we used one red and 12 greens. For Thursday, we used one red and 18 greens, and for Friday, we used one red and 24 greens.



Our total was 5 red blocks and 60 green blocks. Since Angela ate a total of 100 grapes, the 5 red blocks had to stand for  $100 - 60$  grapes or 40 grapes. If we traded the 5 red blocks for 40 green (individual grape) blocks, we could make 5 piles of 8 green blocks. So, that means that Angela ate 8 grapes on Monday.

We checked our work by starting on Monday with 8, added 6 more each day and then added those numbers together:

$$8 + 14 + 20 + 26 + 32 = 100$$

Our answer checks!

### Method 3: Guess and Check

I took a guess that Angela ate 10 grapes on Monday. Then I figured out how many she ate on the other four days. 16 on Tuesday, 22 on Wednesday, 28 on Thursday, 34 on Friday.

$$10 + 16 + 22 + 28 + 34 = 110$$

That told me 10 was too many for Monday. Next I tried 9 grapes on Monday and added six more grapes each day:

$$9 + 15 + 21 + 27 + 33 = 105$$

All these numbers are odd as is the total 105. I noticed that changing Monday from 8 to 9 made a difference of 5 in the total. I needed to find a way to get a total that was 5 less and an even number, so I decreased Monday's grapes by one and tried 8:

$$8 + 14 + 20 + 26 + 32 = 100 \text{ grapes}$$

### Method 4: Direct Approach

I counted up all the 6s that would be added. One 6 on Tuesday, 2 more 6s on Wednesday, 3 more on Thursday, 4 more on Friday. That totals 10 sixes, or 60.  $100$  total grapes -  $60 = 40$  grapes, which Angela ate in equal portions over five days.  $40 / 5 = 8$ . Angela must have eaten 8 grapes the first day.

### Method 5: Algebra

I let  $x$  represent the number of grapes Angela ate on Monday. On Tuesday she ate  $x + 6$ , on Wednesday  $x + 2(6)$ , Thursday  $x + 3(6)$ , Friday  $x + 4(6)$ .

$$5x + 10(6) = 100$$

$$5x + 60 = 100$$

$$5x = 40$$

$$x = 8$$

Angela ate 8 grapes on Monday.

**Extra:** Starting on Saturday, I kept adding 6 new grapes to the number of grapes Angela ate the day before. I kept track of the totals until the total became higher than 300. It happened on Wednesday, the tenth day, when the total reached 350.

	Grapes Eaten	Total
Saturday	38	138
Sunday	44	182
Monday	50	232
Tuesday	56	288
Wednesday	62	350

### Standards

If your state has adopted the [Common Core State Standards](#), you might find the following alignments helpful.

#### *Grade 3: Operations & Algebraic Thinking*

Solve problems involving the four operations, and identify and explain patterns in arithmetic.

#### *Grade 3: Number & Operations in Base Ten*

Fluently add and subtract within 1000 using strategies and algorithms based on place value, properties of operations, and/or the relationship between addition and subtraction.

#### *Grade 4: Operations & Algebraic Thinking*

Use the four operations with whole numbers to solve problems.  
Generate and analyze patterns.

#### *Grade 4: Number & Operations in Base Ten*

Fluently add and subtract multi-digit whole numbers using the standard algorithm.

#### *Grade 5: Operations & Algebraic Thinking*

Analyze patterns and relationships.

#### *Mathematical Practices*

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision

## Teaching Suggestions

This problem is one of my all-time favorites! I've used it with elementary students, middle school students, pre-service teachers, in-service teachers, and with each audience I introduce the problem by saying, "I'm going to tell you a story." This problem just seems to work great as a "read-aloud." I put on my dramatic story-telling voice and announce the title, "Eating Grapes." I read the first paragraph and stop short of asking the question "How many grapes did she eat on Monday?" and instead I pause, look out at the audience, and ask "What did you hear?" It's such a non-threatening question.

I call on students/folks until there are no more things for them to tell me. As they are responding, I don't repeat and I don't react. They're simply telling me what they "heard." If you are familiar with our "I Notice/I Wonder" activity, you'll recognize this as an oral version of it.

Next I announce that I'll read the "story" again and before I read I ask everyone to be thinking about what was said first. I ask them to consider if the responses were accurate or if any of them should be adjusted. This activity increases the chances that the listeners will have a better grasp of what is happening in the problem.

When we first offered Eating Grapes, many students interpreted "six more grapes than the day before" as merely "six more grapes." We also noticed that students thought that Angela ate 100 grapes on Friday alone. It seemed to us that these errors might stem from difficulty in reading comprehension or from jumping to conclusions and failing to reflect on whether the final answer works in the original problem. Either the "read-aloud" version of "I Notice/I Wonder" or another version of it could definitely help with interpretation because it provides the time for students to make sense of the problem and communicate their thinking before working on finding an answer.

### Sample Student Solutions - Focus on *Interpretation*

In the solutions below, I've provided scores the students would have received in the **Interpretation** category of our scoring rubric. My comments focus on areas in which they seem to need the most improvement.

Novice	Apprentice	Practitioner	Expert
Understands few of the criteria listed in the Practitioner column.	Understands most but not all of the criteria listed in the Practitioner column.	Understands that <ul style="list-style-type: none"><li>the goal is to find how many grapes Angela ate on Monday.</li><li>the total number of grapes eaten over five days (Monday through Friday) is 100.</li><li>the number of grapes eaten each day increases by 6 per day.</li></ul>	Is at least a Practitioner in Strategy and comes up with the correct solution for the Extra.

#### Eric, age 12, Novice

Angela ate 4 grapes on Monday.

All I did was 6 divided by 100 hundred and I found the remainder was the number of grapes that angela ate on monday.

*Eric lacks understanding of the key concepts. I'd ask him to start by just noticing. This might get him to think about the problem rather than just using an operation and some numbers to try to quickly find an answer.*

**Grace, age 11, Novice**

Angela ate 70 grapes on Monday.

The strategy I used was multiplying and subtracting. First, I multiplied 6 grapes by the 5 days from Monday to Friday to get the number of grapes that she didn't eat on Monday. I got 30. Then I subtracted 30 (the number of grapes she didn't eat on Monday) from 100 to get 70, my final answer.

*Grace writes a fairly detailed explanation of what she did and why. I'd ask, "Did you remember that Angela eats six more grapes each day than she ate the day before?"*

**Rodney, age 13, Apprentice**

On Monday Angela ate 76 grapes.

Since we know that on Friday she ate a total of 100 grapes, I just work backwards from there.

Since she ate 100 on Friday, that means she ate 94 on Thursday. Since she ate 94 on Thursday, she ate 88 on Wednesday. 88 on Wednesday, 82 on Tuesday. 82 on Tuesday, 76 on Monday.

*Rodney understands that Angela eats six more grapes each day than the previous day. He thinks she ate 100 grapes on Friday alone. I'd ask him to read the last sentence of the problem and paraphrase it. Did you understand that 100 is the total of all the grapes she ate over five days?*

**Alyssa, age 9, Apprentice**

She ate 76 grapes on Monday.

First I multiplied 6 times 4. to find out the total of grapes she ate Tuesday through Friday. Then I subtracted the 24 I got for the answer from one hundred [How many she ate in the five days.] I got 76 grapes total. That's how I figured out she ate 76 grapes.

*Alyssa arrives at the same answer as Rodney. She understands that 100 is the total for the week, but thinks that Angela eats only 6 grapes on each of the 4 days after Monday. I'd ask her what she thinks the second sentence of the problem means.*

**Tim, age 11, Practitioner**

She ate eight grapes, on Monday.

First I knew that if I did six grapes on Monday, that I would get six \* 15. Which equals 90. I knew that I would need ten more. So I tried ten. Then I got 110. Then I knew I needed ten less. So I tried nine. Then I got 105. So I thought that every time I go down by one my total goes down by five. So since I needed 100 I went down by one more to 8. I began with 8 and added 6 to it to get 14 and continued to add 6 three more times until I got to my answer of 100.

*Tim uses good number sense in testing 6 by clustering the week's worth, 15 groups of 6 or  $6 * (1+2+3+4+5)$ , although he needs to explain why. He adjusts his trials thoughtfully. I'd like him to show his other test calculations.*

**Jordan, age 9, Practitioner**

Angela ate 8 grapes on Monday.

First, I made a chart and put down 6 grapes on Monday, 12 on Tues., 18 on Wed., 24 on Thurs., and 30 on Fri. Then I added it all up. The sum was 90. After that, I realized that if I added another grape for Monday and did a pattern adding 6 every day there would be 5 more grapes in the sum. Technically speaking, it's a matter of multiplication: 1 grape X 5 days. So that means, if I added 2 grapes to Monday and did the add 6 pattern I'd get the answer. (100 grapes on Friday) This is what the correct list looked like: Mon. 8 grapes, Tues. 14 grapes, Wed. 20 grapes, Thurs. 26 grapes, Fri. 32 Grapes.

*Jordan gets great mileage from his first guess. He uses what he learns to find the answer directly. I'm glad he shows his first and his final lists of numbers. I wonder what prompted him to start with 6*

### Michael, age 12, Practitioner

On Monday, Angela ate 8 grapes.

I drew out a timeline, Monday-Friday, picked a logical number for Friday and subtracted 6 from that number for each day and added them together. I repeated the process until the total came to 100.

*Michael's clever guess-and-test begins with Friday and works backward. To make his solution complete, he needs to show some of his tests and tell how he adjusted his guesses.*

### Megan, age 11, Practitioner

The total number of grapes Angela ate on Monday was 8 grapes.

First I substituted the number of grapes Angela ate on Monday with  $x$ . Then for Tuesday I put  $x+6$ , Wednesday  $x+12$  because you are adding 6 Tuesday then 12 Wednesday then Thursday you have to add  $x+18$  because since the number  $x$  is representing isn't changing yet because we don't know what  $x$  represents you just add six to the number you added before. Friday you would add  $x$  and 24. Then you have to add the numbers up.  $6+12+18+24=60$ . To find out how many grapes Angela ate Monday you have to do  $60+(5 \times \_) = 100$ . To find what blank equals which is the number of grapes Angela ate on Monday you have to divide 40 by 5 because if you subtract 60 from 100 you get 40. The answer to the division problem is 8, the number of grapes Angela ate on Monday.

*I doubt that Megan has been introduced to formal algebra, but she exhibits good algebraic thinking. It's always a good idea to verify algebraic solutions by testing the numbers in the original problem. A list of the grapes eaten each day would do that.*

### Paul, age 11, Expert

Angela ate 8 grapes on Monday.

She ate 8 grapes on Monday. She ate 8 because each day after Monday she ate 6 more grapes than the day before. So on Tuesday she ate 14 grapes ( $8+6=14$ ). On Wednesday she ate 6 more grapes than she did on Tuesday so she ate 20 grapes because  $14+6=20$ . So far she's eaten 42 grapes. On Thursday she ate 6 more grapes than she ate on Wednesday so she ate 26 grapes because  $20+6=26$ . So far she's eaten 68 grapes. On Friday she ate 6 more grapes than Thursday so she ate 32 grapes because on Thursday she ate 26 grapes and  $26+6=32$ . Since Friday was the last day she has eaten 100 grapes because if you add up all the numbers (8,14,20,26,32) you get 100.

She would eat her 300th grape on the 10th day because in 5 more days she'll have eaten 350 grapes which is more than 300. She would eat her 300th grape on the 10th day because if she keeps eating 6 more grapes each day on the 6th day she'll eat 38 grapes, the next day 44 grapes, the day after that 50 grapes, then 56 grapes, and finally on the 10th day she'll eat 62 grapes. If you add it all up you'll get 350. It has to be 350 and not 300 because if you subtract one day, the last day which is 62 grapes, she'll only have eaten 288 grapes which is less than 300.

*Paul does a fine job of verifying his answer and convincing us that he is correct. Since he fails to explain how he found his answer, his solution is not complete, but he does show good understanding of the key math ideas of the problem. That, along with his demonstrated grasp of the Extra earns him Expert in Interpretation.*

*To encourage him to improve his Completeness score, I'd ask how he arrived at 8 in the first place.*

## Scoring Rubric

A **problem-specific rubric** can be found linked from the problem to help in assessing student solutions. We consider each category separately when evaluating the students' work, thereby providing more focused information regarding the strengths and weaknesses in the work.

We hope these packets are useful in helping you make the most of Math Fundamentals Problems of the Week. Please let me know if you have ideas for making them more useful.

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