NATIONAL COUNCIL OF TEACHERS OF MATHEMATICS

Problem of the Week Teacher Packet

Emmy's Action Figures

Emmy is collecting Famous Mathematician action figures. So far she has one Grace Hopper figure, one Leonardo Fibonacci, and two identical Leonhard Eulers that look exactly alike. Emmy keeps them in a straight line on a shelf above her desk and likes to rearrange them each week. How many different-looking arrangements can she make before she must repeat one?

Explain how you found your arrangements, and how you can be sure you have found all of them.

Extra: In how many of Emmy's arrangements is Leonhard Euler at the left end? In how many arrangements is Grace Hopper at the left end? Describe and explain any difference you notice.

Answer Check

After students submit their solution, they can choose to "check" their work by looking at the answer that we provide. Along with the answer itself (which never explains how to actually get the answer) we provide hints and tips for those whose answer doesn't agree with ours, as well as for those whose answer does. You might use these as prompts in the classroom to help students who are stuck and also to encourage those who are correct to improve their explanation.

Emmy can make 12 different arrangements of her Famous Mathematician action figures.

If your answer **doesn't** match ours,

- did you realize that merely swapping the positions of the two Euler figures does not create a different-looking arrangement?
- did you try using objects and moving them around to create different arrangements?
- did you make an organized table to help keep track of the arrangements?
- did you look for patterns?

If any of those ideas help you, you might revise your answer, and then leave a comment that tells us what you did. If you're still stuck, leave a comment that tells us where you think you need help.

If your answer **does** match ours,

- is your explanation clear and complete?
- did you try the Extra question?
- did you verify your answer with another method?

Revise your work if you have any ideas to add. Otherwise leave us a comment that tells us how you think you did—you might answer one or more of the questions above.

Our Solutions

Method 1: Notice/Wonder

Our group used the Activity Series worksheet that our teacher gave us and we listed our noticings and wonderings.

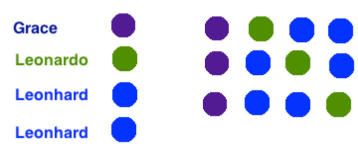
Here is our list of noticings:

- Emmy is a collector.
- She collects action figures.
- She has four figures.
- One of her figures is called Grace Hopper.
- One of her figures is called Leonardo Fibonacci.
- Two of her figures have the same name and it's Leonhard Euler.
- Emmy has her four figures on a shelf in line.
- She likes to move them around.
- Each person in the picture had on some kind of hat.

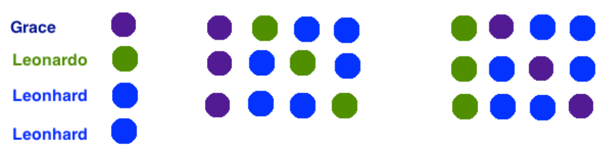
We wondered:

- who those three people were in the photos.
- why they each wore a hat.
- why she moved them.
- if she moved them using some sort of system.

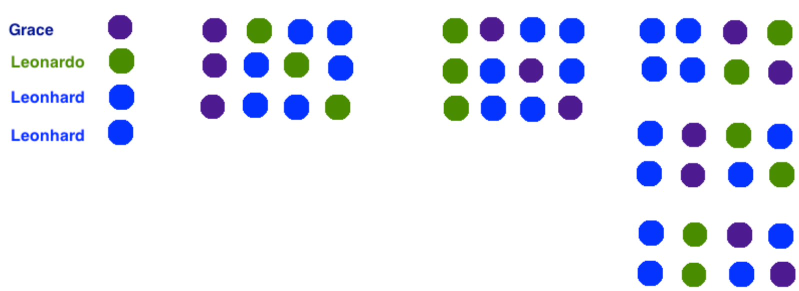
We started to see if we could find out how many ways she might move them and we started to think about it by drawing a picture. We used colors and thought about Grace being first and how that might work:



We noticed we could see a pattern. Next we thought about how it would look with Leonardo first. We added to our picture:



Again we saw a pattern. Next we thought about one or both of the Leonhards being first. We added to our picture:



We double-checked that we'd tried all the arrangements and that we didn't have any duplicates. We found 12 ways Emmy could arrange her action figures.

Method 2: Use Manipulatives to Act It Out

Right away (when I saw this problem) I grabbed 4 of my stuffed animals. Then I cut out 4 little squares of paper. I wrote G on one card, L on another, and since there are 2 Eulers I used E1 and E2. I taped 1 name to each stuffed animal and started out in the order of appearance. (G first, then L, etc.) I actually made each arrangement, step by step, and I wrote them down. But when I was done, I realized E1 and E2 looked exactly alike. So I crossed out the double when E1 and E2 appear in the same place in the arrangement.

GLE1E2
~~GLE2E1~~ double
GE1E2L

GE1LE2
~~GE2LE1~~ double
~~GE2E1L~~ double

LGE1E2
~~LGE2E1~~ double
LE2GE1
LE2E1G
~~LE1GE2~~ double
~~LE1E2G~~ double

E1LGE2
E1GLE2
E1GE2L
E1E2LG
E1E2GL
E1LE2G

~~E2GE1L~~ double
~~E2E1GL~~ double
~~E2E1LG~~ double
~~E2GLE1~~ double
~~E2LGE2~~ double
~~E2LE1G~~ double

Emmy can arrange the figures 12 different ways.

Method 3: Use a Table

We used a table to list all the ways that Emmy could arrange her figures.

H (Hopper) is first	F (Fibonacci) is first	E (Euler) is first
HFEE	FHEE	EFHE
HEFE	FEHE	EFEH
HEEF	FEEH	EHFE
		EHEF
		EEFH
		EEHF

We counted the ways and there are 12.

Method 4: Make a List

I made a list of all the ways Emmy could arrange the figures. I used
E for Euler
H for Hopper
F for Fibonacci

My list started with all the ways Fibonacci could be first in line.
FHEE
was my first way. Then I moved H one place to the right:
FEHE
Then I moved H one more place to the right:
FEEH
If H has been in all three possible places, and the others positions are Es that look alike, I know I have found all the ways F can be first.

I used the same system for H in the first place.
HFEE
HEFE
HEEF

Then I listed the ways that an E could be first. There are two ways EE can be first together, because there are two ways F and H can be 3 and 4.

EEFH
EEHF

Similarly, there are two ways that EF can be first:

EFEH
EFHE

And two ways that EH can be first:

EHFE
EHEF

That makes 12 ways Emmy can arrange the figures.

Extra: I looked at my list of 12 arrangements and found 6 that had Leonhard Euler on the left end and 3 that had Grace Hopper on the left. I noticed that there were twice as many Eulers as Hoppers. That is because with Euler on the left, Emmy has three different figures to arrange in the 2nd, 3rd and 4th positions. With Grace Hopper on the left, Emmy has only 2 different-looking figures for positions 2, 3 and 4.

Standards

If your state has adopted the Common Core State Standards, you might find the following alignments helpful.

Grade 4: Operations & Algebraic Thinking

Generate and analyze patterns.

Grade 5: Operations & Algebraic Thinking

Analyze patterns and relationships.

Mathematical Practices

1. Make sense of problems and persevere in solving them.
3. Construct viable arguments and critique the reasoning of others.
7. Look for and make use of structure.

Teaching Suggestions

Solvers of ***Emmy's Action Figures*** need to find all the ways that four action figures can be arranged in a straight line. Since two of the figures are identical, and the problem asks for "different-looking arrangements," solvers need to realize that merely switching the position of the two Eulers does not create a new arrangement. The key to success is using a system that allows a solver to find all arrangements without repeating. The methods that children use for finding the arrangements and their record keeping systems will vary, but all solutions should demonstrate a systematic approach.

Sample Student Solutions – Focus on *Strategy*

In the solutions below, I've provided scores the students would have received in the **Strategy** category of our scoring rubric. My comments focus on areas in which they seem to need the most improvement.

Novice	Apprentice	Practitioner	Expert
Has no ideas that will lead them toward a successful solution or shows no evidence of strategy.	Uses a strategy that uses luck instead of skill, or doesn't provide enough detail to determine whether it was luck or skill.	Uses a strategy that relies on skill, not luck, which might include: • thorough noticing and wondering • acts it out (possibly using manipulatives • make a list, chart, or table • logical reasoning	Does one or more of these: Uses two different strategies. Uses a good Extra strategy. Uses an unusual or sophisticated strategy, e.g., effective and appropriate use of technology.

Shira, age 10, Novice

She can make 16 arangments before she has to repet one of them.

I timed 4 times 4 and because there is only 4 action figures and got 16. That is my answer. 16.

I notice that Shira multiplied but I'm not too sure why.

I might ask her if she tried to act out the problem and talk with a partner as she tried some of the different arrangements. What might help to keep track of the possibilities?

Shaina, age 10, Novice

Emmy has 12 different looking arangements before she has to repeat.

Emmy has 4 places on her shelf. She has 3 DIFFERENT action figures.4 spaces times 3 figures equals 12 different looking arangments on the shelf.

I notice that Shaina got the correct answer but it's interesting what happens when you try this counter-example:

What if Emmy has four identical looking action figures. Using her method "4 spaces times 1 figure" equals 4 different looking arrangements and yet it should only be 1. I might ask her to try her method with 3 (or 4) identical action figures and see if she notices that it doesn't work.

Michael, age 9, Novice

Well there is 3 answers i think thats what i found.

Well first i got 3 answers i got all the answers i could. I wrote some down on a piece of paper and found as many ways i as i could here was what i got leonardo/gracehopper. leonhard/gracehopper. leonardo/leonhard.

I notice Michael has the idea of writing arrangements down on paper. I'm wondering why his 3 answers only have 2 action figures noted at a time. I wonder if he realized that his arrangement should include all 4 figures.

Because he's not given too much of an explanation he may have a more developed *strategy than I've given him credit for. I would encourage him use a manipulative of some kind to continue thinking about the possible arrangements.*

Jared, age 12, Apprentice

There are ten different possibillities.

First, I abreviated every action figure's name. Then, I put them in different orders.

Here they are: LE LF LE GH, LE LE LF GH, LE GH LF LE, LE LE GH LF, LF LE LE GH, LF LE

GH LE, LF GH LE LE,GH LE LE LF, GH LE LF LE and GH LF LE LE.

I notice Jared used two letters to represent each action figure. I would suggest using only one letter (perhaps just the initial of the last name). I might also suggest he use a table or a vertical list to organize his arrangements. It might help him see if he's missing anything.

A., age 11, Practitioner

I got 12 as my answer to how many ways can Emmy put her action figures. before she repeats any.

I put each action figure at the first spot and kept switching the middle around until I had every combination. Then I moved another

into the first spot and did the same thing for each letter.

e=Leonhard Eulers
l=Leonnardo Fibonacci
g=Grace Hopper

glee
gele
geel
leeg
lege
lgee
egel
eegl
eelg
eleg
elge
egle

A. has used a strategy that exhibits skill and not just luck. I might ask why the "middle" was the only thing switched around. That might prompt A. to add just a little more to the explanation to improve the Completeness score.

Adrienne, age 10, Expert

My answer for emmy's action figures is 12 ways. EXTRA: The answer to the extra is 3 times for grace hopper and 6 times for leonhard Eulers

First, I made a organized list .On the top of the organized list was first,second, third, foruth.To show the order i made grace hopper=gh, Leonard Eulers= le, Leonardo Fibonacci=lf. Then i put the intasils in the colums. The organized list i used looked like this and this is yhe way i found out that it was 12 ways.

first	second	third	fourth
gh	lf	le	le
gh	le	lf	le
gh	le	le	lf

Adrienne has chosen a strategy that works well both for her main problem but also the Extra.

I would suggest she reflect and revise to improve her Clarity score.

lf	le	le	gh
lf	le	gh	le
lf	gh	le	le
le	gh	lf	le
le	le	lf	gh
le	lf	gh	le
le	gh	le	lf
le	le	gh	lf
le	lf	le	gh

extra: On my organized list i checked off how many times LE and GH were shone. LE shod up 6 times because there were two LE's. For GH it shod up 3 times because there is only one GH. The place it shod up in was in the first place. The difference i noticed was that the was more LE's because there was two. Also there was only one GH.

Scoring Rubric

A **problem-specific rubric** can be found linked from the problem to help in assessing student solutions. We consider each category separately when evaluating the students' work, thereby providing more focused information regarding the strengths and weaknesses in the work.

We hope these packets are useful in helping you make the most of the Math Fundamentals Problems of the Week. Please let me know if you have ideas for making them more useful.

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