Math Fundamentals PoW Packet
Sharing Smelt
Problem 4615   ·   https://www.nctm.org/pows/

Welcome

This packet contains a copy of the problem, the "answer check," our solutions, some teaching suggestions, and samples of the student work we received in January 2008. The text of the problem is included below. A print-friendly version is available using the "Print" link on the problem page.

Standards

In *Sharing Smelt* students are asked to determine how many smelt Larry caught Saturday morning. The **key concepts** are addition, subtraction, and halving and doubling of whole numbers.

If your state has adopted the Common Core State Standards, this alignment might be helpful.

Grade 3: Operations & Algebraic Thinking

Solve problems involving the four operations, and identify and explain patterns in arithmetic.

Grade 4: Operations & Algebraic Thinking

Use the four operations with whole numbers to solve problems.

Grade 5: Operations & Algebraic Thinking

Analyze patterns and relationships.

Mathematical Practices

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.

The Problem

Sharing Smelt

Larry spent Saturday morning ice fishing for smelt. He caught a bucketful! On the way home he stopped for lunch at his friend Daryl's house. He gave Daryl half of his catch plus 3 more fish.

When he saw his mother at the post office, he gave her half of his remaining fish plus 3 more.

Larry then went to his brother Calvin's house to watch a game on Calvin's large screen TV. They fried up half of the remaining smelt plus 3 more for a snack.

When Larry got home, he looked in his bucket and found only 10 smelt left, which he saved for breakfast.

How many fish did Larry catch?

Explain how you found your answer. Show how you know it is correct.

Extra: What if Larry found 20 smelt left in his bucket? Or 30? What can you say about the starting number of fish whenever the final remainder is a multiple of 10?

Smelt are a small fish caught in lakes and rivers of Canada and northern United States. You can read more about them at http://en.wikipedia.org/wiki/Smelt.

After students submit their solution, they can choose to "check" their work by looking at the answer that we provide. Along with the answer itself (which never explains how to actually **get** the answer) we provide hints and tips for those whose answer doesn't agree with ours, as well as for those whose answer does. You might use these as prompts in the classroom to help students who are stuck and also to encourage those who are correct to improve their explanation.

Larry caught 122 fish on Saturday.

If your answer **doesn't** match ours,

- did you try working backwards?
- did you make a table to help keep track of what happened?
- did you look for patterns?
- did you check your arithmetic?

If you used guess and check, did you tell . . .

- what numbers you tried?
- how you checked them?
- how you knew whether they worked or not?
- how you decided what to try next?
- about any patterns that helped you?

If any of those ideas help you, you might *revise* your answer, and then leave a comment that tells us what you did. If you're **still stuck**, leave a *comment* that tells us where you think you need help.

If your answer **does** match ours,

- is your explanation clear and complete?
- did you try the Extra questions?
- did you verify your answer (show how you know you have found them all)?

Revise your work if you have any ideas to add. Otherwise leave us a *comment* that tells us how you think you did—you might answer one or more of the questions above.

Method 1: Work Backwards

I worked backwards from the end of the story. After sharing 1/2 of his remaining fish plus 3 more with Calvin, Larry had 10 smelt left. That means half of the fish must have been 10 + 3 = 13. There must have been twice that, or 26, in the bucket before he arrived at Calvin's house.

Those 26 represent 3 less than half of what Larry had before he gave his mother 1/2 plus 3 more.

26 + 3 = 29 [half of what he had]

2 · 29 = 58 fish before giving some to his mother

Those 58 are 3 less than half of what he caught, before stopping at Daryl's house.

58 + 3 = 61 and 2 · 61 = 122 fish

Larry caught 122 smelt.

I checked my answer by working forwards.

Larry gave 1/2 of 122 fish plus 3 more to Daryl.

1/2 of 122 = 61 and 61 - 3 = 58, the number of smelt Larry had left after giving some to Daryl.

1/2 of 58 = 29 and 29 - 3 = 26, the number of smelt he had after giving some to his mother.

1/2 of 26 = 13 and 13 - 3 = 10, the number of smelt he had after sharing with Calvin.

Extra: I figured out a pattern. Starting with the final number, add 3 and double it to get the number of fish in the bucket before the previous step. That reverses what happened when Larry gave away half the fish plus 3 more. Repeat it three times. So for a final number of 20:

20 + 3 = 23 and 2 · 23 = 46

46 + 3 = 49 and 2 · 49 = 98

98 + 3 = 101 and 2 · 101 = 202 total fish in Larry's catch

For a final number of 30:

30 + 3 = 33 and 2 · 33 = 66

$66 + 3 = 69$ and $2 \cdot 69 = 138$

$138 + 3 = 141$ and $2 \cdot 141 = 282$

When the final number is a multiple of 10, the starting number has a 2 in the ones place.

Method 2 – Systematic Guess and Check

I knew that Larry must have started with an even number of fish before each stage, since he always divided them in half. I knew he must have started with at least 80 since 1/2 of 18 = 40, 1/2 of 40 = 20 and 1/2 of 20 = 10. 80 didn't work, as he wound up with an odd number after the first round, so I tried the next even number. 82 worked all the way, but only left 5 in the barrel, so I went up by 8 (three doublings of 2). That increased the final number by one fish. I added four more 8s (32) to 90 and tried 122.

starting # of smelt	gave Daryl 1/2 + 3	remaining fish	gave mom 1/2 + 3	remaining fish	ate w/ Calvin 1/2 + 3	remaining fish
80	43	37 (odd)				
82	44	38	22	16	11	5
90	48	42	24	18	12	6
122	64	58	32	26	16	10

Larry started with 122 smelt.

[Note: This method has a built-in check by showing each stage of the story. Students who use guess-and check could be challenged to use what they've learned to construct an alternate solution strategy by working backwards. They might also be encouraged to try the Extra, since this kind of record keeping lays the foundation.]

Method 3 – Pre-Algebra, using expressions

I worked backwards. I made algebraic equations, using x for the starting number of fish, y for the remaining fish after Larry gave some to Daryl, and z for the remaining fish after giving some to his mother.

$(1/2)z - 3 = 10$
    Larry shared 1/2 the remaining fish plus three more with Calvin and wound up with 10 left.

$(1/2)z = 13$ $z = 26$ fish remaining after giving some to his mother

$(1/2)y - 3 = 26$
    Larry gave 1/2 the remaining fish plus three more to his mother and wound up with 26 left.

$(1/2)y = 29$ $y = 58$ fish remaining after giving some to Daryl

$(1/2)x - 3 = 58$
    Larry shared 1/2 of his total fish plus three more with Daryl and wound up with 58 left.

$(1/2)x = 61$ $x = 122$ starting number of fish

[Note: Algebraic approaches should be verified by substituting the solution into the original problem, as in Strategy 1.]

Extra: To find starting numbers with 20 as the final number, substitute 20 for 10 in the first step above.

       $z = 46$ $y = 98$ $x = 202$

Similarly, for 30 as the final number, substitute 30 for 10 in the first step.

       $z = 66$ $y = 138$ $x = 282$

For each increasing multiple of 10 as the final number of fish, the starting number increases by 80.

Teaching Suggestions

Acting out (modeling) the problem with manipulatives (Goldfish crackers or gummy fish, or even boring base blocks), whether forwards or backwards, may help students conceptualize what is happening and discover a pattern that leads to more generalized operational rules. If your students are familiar with the idea of function machines, they can think of this as a machine that "eats" one-half of the input number plus 3 more. The output number becomes the input for the second and third rounds.

For students who work backwards, the problem could be an opportunity to discuss inverse operations. Subtraction reverses the process of addition, and doubling reverses the process of halving. It's

important to note that the order of the operations must also be reversed. In working forward halving takes place before subtracting 3 more. In reverse, the 3 must be replaced before doubling. When getting dressed, socks go on before shoes. At the end of the day, the shoes come off before the socks!

Sample Student Solutions

focus on **Interpretation**

In the solutions below, we've provided the scores the students would have received in the **Interpretation** category of our scoring rubric. Our comments focus on what we feel is the area in which they need the most improvement.

Novice	Apprentice	Practitioner	Expert
Understands few of the criteria listed in the Practitioner column.	Understands most but not all of the criteria listed in the Practitioner column.	Understands that • the question asks how many fish Larry caught. • Larry gives up half of his remaining smelt plus 3 more fish, each of three different times. • Larry has 10 fish left after those events.	Is at least a Practitioner in Strategy and comes up with the correct solution for the Extra.

Owen

age 9

Interpretation
Novice

my answer is 89.

I started from the end of the problem to the beginning

Although Owen hasn't provided much explanation, I would suggest that he try his strategy.

I wonder what he did as he "… started from the end…." Was there a number? What did he do with it?

Emily

age 12

Interpretation
Novice

larry caught 50 fish

i solved this problem by creating an algibra problem for my self the I just solved it.

Like Owen, Emily has mentioned a reasonable strategy.

I wonder if she can tell me the "algebra problem" she created.

John
age 11

Interpretation
Apprentice

Larry caught 110 smelt.

First, I randomly guessed numbers. Second, I divided and subtracted the numbers used in the method. Lastly, I figured out what numbers to use and then I answered the problem.

I notice that John mentions guessing numbers but I wonder if he used the other part of that strategy.

I might ask his if/how he "checked" his guesses. I'd also suggest that he include some of the numbers and calculations.

Sylvia
age 12

Interpretation
Apprentice

Larry caught 89 fish.

The answer to to the problem would be 89 because in the end there where 10 fish left. Ten is half of 20, 20 is half of 40,and 40 is half of 80. Including the three fish that he gave to each of the people the answer would be 89.

I notice that Sylvia has noticed the pattern of halving but she doesn't include the "plus 3 more" at each step. Instead she adds nine at the end.

I wonder if she thought about whether or not there was a difference, or if she is simply misunderstanding that the 3s would be affected by the halving. I would suggest that she try it "forward" using the 89 and see what happens.

Madison
age 11

Interpretation
Practitioner

Larry caught 122 fish.

Larry went fishing and caught 122 fish. I figured out the answer by trying many different numbers and making sure I knew the facts. The first number I tried was 30. I figured out that at Daryl's house he gave Daryl 18 fish. When he saw his mom he gave her 9 fish. He was left with 6 fish and you know he is going to have to divide it in half again and give another three fish as well. He is going to have 10 fish when he gets home so that can not be right. I kept on trying out numbers until I realized something. It would be a lot easier if I worked backwards.

So I worked backwards and started at the part about Larry having 10 fish when he went home. Since there was nothing to do there I went on and did the part about Calvin's house. Since they fried up half and ate three more I thought about the 10 fish he had when he got home and realized it was 3 less than half. So 10+3=13 or half the fish so he had 26 fish when he got to Calvin's house. Then 26 fish is 3 less than half the fish he gave his mom. 26+3=29 or he had 58 fish when he left Daryl's house. 58+3=61 or half the fish he gave Daryl. 61x2=122 fish caught.

I notice Madison tried a Guess and Check strategy and then decided to change to Working Backwards. Including both ideas in her explanation, Madison gives the reader a window into her thinking.

I would suggest that she try the Extra next!

Ben

age 11

Interpretation
Practitioner

Larry caught 122 smelts while ice fishing.

I reversed the question and here's how I did it:

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 10 smelts when he got home
+ 3 because he gave Calvin three more then half
 13
x 2 because he gave half of his remaining bucket to Calvin
 26 smelts before he went to Calvin's house
+ 3 because he gave Mom three more then half
 29
x 2 because he gave half of his remaining bucket to Mom
 58 smelts before he met his mom  at the post office
+ 3 because he gave Daryl three more then half
 61
x 2 because he gave half of his remaining bucket to Daryl
122 Before he met Daryl
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            122 is my final answer

Ben, like Madison, uses a "work backwards" strategy but in a more word-efficient way of explaining his thinking.

I like his use of "because" to explain his thinking each step along the way.

I would encourage him to try the Extra.

Liam

age 13

Interpretation
Expert

Larry caught 122 fish on Saturday morning. BONUS: Every time Larry has another 10 smelt left in his bucket, he has to add 80 more to his starting number of fish he catches.
Daryl: $1/2x - 3 = y$
Mom: $1/2y - 3 = z$
Calvin: $1/2z - 3 = 10$

I realized that this problem could be worked backwards. So I started with Calvin:

$1/2z - 3 + 3 = 10 + 3$
$1/2z = 13$
$2(1/2z) = 13(2)$
$z = 26$

Larry had to have 26 smelt when he arrived at Calvin's house. He would cook $13 + 3 = 16$ fish and still have 10 in his bucket for breakfast.

Then I solved for Larry's mom using the 26 as z (the answer).

$1/2y - 3 + 3 = 26 + 3$
$1/2y = 29$
$2(1/2y) = 29(2)$
$y = 58$

Larry had to have 58 fish when he saw his mom at the post office.
He would give her $29 + 3 = 32$ fish and have 26 left over for when he arrived at Calvin's.

Then I solved for Daryl using the 58 as y (the answer).

$1/2x - 3 + 3 = 58 + 3$
$1/2x = 61$
$2(1/2x) = (2)61$
$x = 122$

Larry had to have 122 fish when he stopped at Daryl's for lunch. He would give Daryl $61 + 3 = 64$ fish and have 58 left for when he met his mom in the post office.

I notice that Liam has interpreted the problem well, including the Extra. He has used a good strategy, combining algebra with working backwards.

His use of paragraph breaks after each equation's solution raises his Clarity but also Reflection score. Those breaks serve to both check and interpret the result and also aid the reader.

One challenge for Liam might be to ask him if he can find a formula that takes e, the number of fish at the end, and calculates x, the starting number of fish.

BONUS: If you start at the end with 20 fish in Larry's bucket, he needs 46 fish when he gets to Calvin's house (20 more than if he only had 10 in his bucket.

If you use 46 fish as his y answer when he meets his mom, he needs 98 fish in his bucket to meet his mom (which is 40 more than if he started with 10).

If you use 98 as his x answer when he goes to Daryl's house, he needs 202 fish in his bucket to go to Daryl's house (which is 80 more than if you started with 10).

To make sure this was true, I also calculated to 30 left in his bucket, and saw that inceasing the bucket by 10 again made him need to increase the beginning total by 80 again, the middle total by 40, and the last total by 20 (he needed to start with 282 fish, have 138 when he meets his mom, and 66 when he gets to Calvin's house).

Scoring Rubric

A **problem-specific rubric** can be found linked from the problem to help in assessing student solutions. We consider each category separately when evaluating the students' work, thereby providing more focused information regarding the strengths and weaknesses in the work.

We hope these packets are useful in helping you make the most of Math Fundamentals Problems of the Week. Please let me know if you have ideas for making them more useful.

https://www.nctm.org/contact-us/