Geometry PoW Packet

Where's Juanita Walking?

Problem 5127 • https://www.nctm.org/pows/

Welcome!

This packet contains a copy of the problem, the "answer check," our solutions, and some teaching suggestions. It does not include samples of student work. The text of the problem is included below. A print-friendly version is available using the "Print" link on the problem page.

Standards

In ***Where's Juanita Walking?***, students use a coordinate grid and counting principles.

If your state has adopted the Common Core State Standards, these alignments might be helpful.

Grade 7: Statistics and Probability

8. Find probabilities of compound events using organized lists, tables, tree diagrams, and simulation. *[In fact, this problem only covers determining the sample space.]*

High School: Statistics and Probability

9. Use permutations and combinations to compute probabilities of compound events and solve problems. *[In fact, this problem only covers determining the sample space.]*

High School: Geometry: Mathematical Practices

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

The Problem

Where's Juanita Walking?

Juanita's city streets are laid out very neatly on a rectangular grid. Her house is two blocks west and two blocks south of City Hall. Her office is three blocks east and one block north of City Hall.

How many different routes could Juanita take from her house to her office, without going out of her way (meaning without ever going south or west)?

Extra: Juanita wondered if there was a quick way to figure out how many different routes she could walk between any two points in the city (still without going out of her way). Can you help her find a formula?

Answer Check

After students submit their solution, they can choose to "check" their work by looking at the answer that we provide. Along with the answer itself (which never explains how to actually **get** the answer) we provide hints and tips for those whose answer doesn't agree with ours, as well as for those whose answer does. You might use these as prompts in the classroom to help students who are stuck and also to encourage those who are correct to improve their explanation.

There are 56 different routes Juanita could take to work.

If your answer **does not** match our answer,

- did you find that her house and her office form a 5 x 3 rectangle on the grid?
- did you try starting with a much smaller rectangle, like 1 x 1, then trying a 1 x 2?
- did you try looking for patterns in the ways you were making paths for the smaller rectangles?

If any of those ideas help you, you might *revise* your answer, and then leave a *comment* that tells us what you did. If you're still stuck, leave a *comment* that tells us where you think you need help.

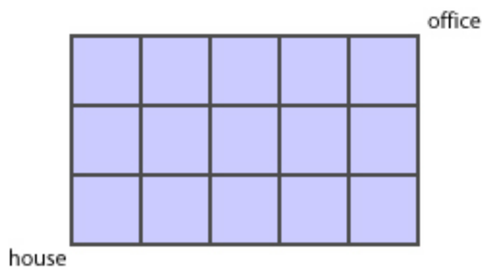
If your answer **does** match ours,

- have you explained how you found your answer, including any formulas you used? (Tell how you knew to use them.)
- did you make any mistakes along the way? If so, how did you find them?
- are there any hints you would give another student?

Revise your work if you have any ideas to add. Otherwise leave us a *comment* that tells us how you think you did—you might answer one or more of the questions above.

Our Solutions

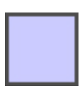
Juanita lives 2 blocks south and 2 blocks west of City Hall. Her office is 3 blocks east and 1 block north of Cith Hall. This will be easier to think about if we ignore City Hall and just say that her office is 5 blocks east and 3 blocks north of her house. So we have a 3 x 5 rectangle that is used for each method shown below.



Method 1: Listing Paths Using Smaller Rectangles and a Table

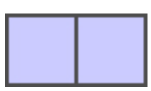
I have to find out how many ways there are to get from her house to her office. It seems like there will be a ton of them, so I'm going to start with smaller rectangles and see if I can see a pattern. I remembered that she can only move up or right, since she doesn't want to backtrack at all and should always be moving towards her office.

1 x 1



She could go up (U) and then right (R), or she could go R and then U. So there are 2 possible ways.

1 x 2

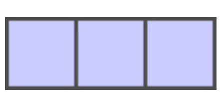


She needs to go up once and right twice.

U R R
R U R
R R U

So there are 3 possible ways.

1 x 3



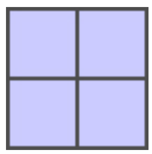
U R R R
R U R R
R R U R
R R R U

There are 4 ways.

I notice that for a 1 x n rectangle, she has to go right n times and up once. There are n + 1 possible places to put the U, so there are n + 1 possible path. I'm going to start a table.

L x W	Paths
1 x 1	2
1 x 2	3
1 x 3	4
1 x 4	5
1 x 5	6

2 x 2



Now I'll move on to the 2 x 2 rectangle. I don't have to do the 2 x 1 rectangle because I already did the 1 x 2 rectangle.

U U R R

U R U R

U R R U

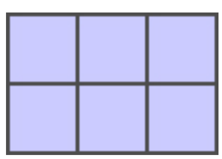
R R U U

R U R U

R U U R

To make sure I had them all, with the first three I put a U, then used the ones from the 1 x 2. Then for the second three, I put an R, then copied the 1 x 2 ones, but swapped the Rs and Us.

2 x 3



I noticed that in the 2 x 2 rectangle, I went Up one, then copied the 1 x 2, then went Right and copied a 2 x 1 So it was like

paths(2 x 1) + paths(1 x 2)

For the 2 x 3, if I go Up, it becomes a 1 x 3. If I go Right, it's a 2 x 2. So we get

paths(1 x 3) + paths(2 x 2) = 4 + 6 = 10

2 x 4

paths(1 x 4) + paths(2 x 3) = 5 + 10 = 15

2 x 5

paths(1 x 5) + paths(2 x 4) = 6 + 15 = 21

I'll add these to the table.

L x W	Paths
1 x 1	2
1 x 2	3

1 x 3	4
1 x 4	5
1 x 5	6
2 x 2	6
2 x 3	10
2 x 4	15
2 x 5	21

I noticed that I could use the rectangles I already had to figure out the new rectangles. I noticed that I was adding the one above in the table to the one with the same second number in the section before. I decided to rearrange the table so that the entries with the same second number are on the same row.

L x W	**Paths**	**L x W**	**Paths**
1 x 1	2		
1 x 2	3	2 x 2	6
1 x 3	4	2 x 3	10
1 x 4	5	2 x 4	15
1 x 5	6	2 x 5	21

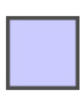
Now I can fill out the rest of the table, knowing that, for example, the 3 x 3 becomes a 2 x 3 and a 3 x 2.

L x W	**Paths**	**L x W**	**Paths**	**L x W**	**Paths**
1 x 1	2				
1 x 2	3	2 x 2	6		
1 x 3	4	2 x 3	10	3 x 3	20
1 x 4	5	2 x 4	15	3 x 4	35
1 x 5	6	2 x 5	21	3 x 5	56

Method 2: Combinations

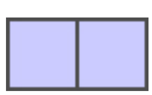
I have to find out how many ways there are to get from her house to her office. It seems like there will be a ton of them, so I'm going to start with smaller rectangles and see if I can see a pattern.

1 x 1



She could go up (U) and then right (R), or she could go R and then U. So there are 2 possible ways.

1 x 2



She needs to go up once and right twice.

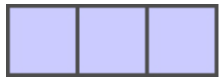
U R R

R U R

R R U

So there are 3 possible ways.

1 x 3



U R R R

R U R R

R R U R

R R R U

There are 4 ways.

I notice that for a 1 x n rectangle, she has to go right n times and up once. There are n + 1 possible places to put the U, so there are n + 1 possible path. I'm going to start a table.

L x W	Paths
1 x 1	2
1 x 2	3
1 x 3	4
1 x 4	5
1 x 5	6

2 x 2

I realized that there were four moves—two U and two R. I had to figure out how many possible ways there were to put 2 U's in 4 slots. This is the same as saying that I have four moves (call them moves 1, 2, 3, and 4). How many ways can I choose 2 of them to be the places where I put the U's? There is a formula for this, where you've got n things and you want to choose r of them:

$$nCr = \frac{n!}{r!\,(n-r)!}$$

So for 4C2, we get

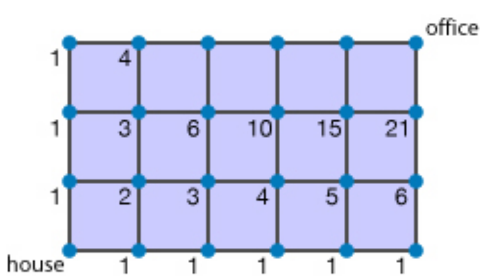
$$\frac{4!}{2!\,(4-2)!} = \frac{4\cdot 3\cdot 2\cdot 1}{2\cdot 1\cdot 2\cdot 1} = 6$$

For the 5 x 3 rectangle, that's 8 total choices, of which 3 are Up.

$$\frac{8!}{3!\,(8-3)!} = \frac{8\cdot 7\cdot 6\cdot 5\cdot 4\cdot 3\cdot 2\cdot 1}{3\cdot 2\cdot 1\cdot 5\cdot 4\cdot 3\cdot 2\cdot 1} = 56$$

Method 3: Pascal's Triangle

It seemed like there were a lot of paths to go 5 blocks East and 3 blocks North, so I started looking at how many ways there are to get to earlier points on her trip. I wrote it on the grid:



There's one way to get to each point on the edge. Then there are two ways to get to the point (1, 1) – you can go through the southern point or the western point.

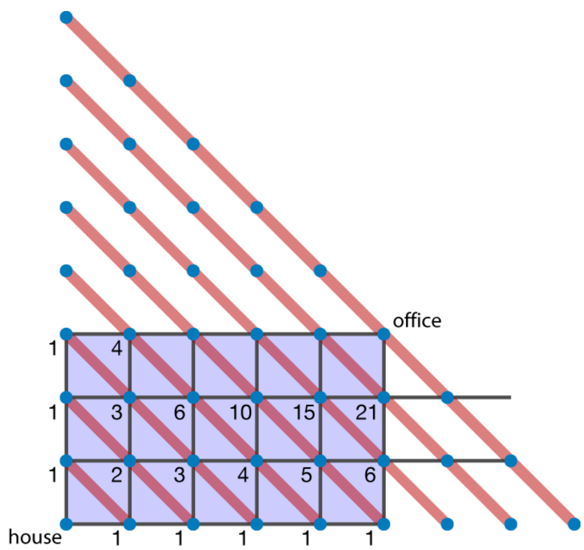
There are three ways to get to the point (3, 2): either you go through (1, 2) or (2, 2), and since there's one way to get to (1, 2), and two ways to get to (2, 2), there are three ways to get to (3, 2).

In general, to find the number of ways to get to a certain point, you add the number of ways to get to the points immediately South and immediately West. I noticed that pattern makes a Pascal's Triangle, lying on its side.

There's a formula to calculate entry x in row y:

$$\frac{y!}{x!\,(y-x)!}$$

So all I have to do is figure out what row and what column her house is in.



Her office is in the eighth row of the triangle, in the third spot (the rows and spots are numbered starting at 0). Notice that it's in the 8th row which is the total number of blocks she has to walk. I can calculate the number of paths using that formula:

$$\frac{8!}{3!\,(8-3)!} = \frac{8 \cdot 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}{3 \cdot 2 \cdot 1 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1} = 56$$

and my final answer is 56 paths.

Teaching Suggestions

This problem may look daunting when you start to dig in, but it's a great chance to practice turning something into a simpler problem (see the Problem Solving and Communication Activity Series document that goes with this problem – you'll find it in the blue box on the problem page). There are many instances in which geometric relationships and drawings are used to tease out ideas in counting and combinatorics. The Handshake Problem (if there are N people in a room and each shakes everyone else's hand exactly once, how many handshakes are there?) is the same problem as calculating the number of diagonals in a polygon. Another problem in our library, *Points, Lines, and Planes*, uses the definition of lines and planes to get students doing some basic counting.

As you can see from our solutions and the strategies suggested in the PS & C document, there are many ways to tackle this. A simple suggestion to start with a smaller rectangle might be all some kids need to get going. They might try a 2 x 2, but I would suggest they start even smaller. Encourage them to pay attention to the ways in which they're keeping track of their guesses and figuring out how many paths – I didn't get this problem until I noticed that I could break each larger rectangle down into smaller rectangles (Method 1 above) that I had already answered, but laying the table out in a specific way helped me figure out the answers more efficiently.

Scoring Rubric

A **problem-specific rubric** can be found linked from the problem to help in assessing student solutions. We consider each category separately when evaluating the students' work, thereby providing more focused information regarding the strengths and weaknesses in the work.

We hope these packets are useful in helping you make the most of Geometry Problems of the Week. Please let me know if you have ideas for making them more useful.

https://www.nctm.org/contact-us/