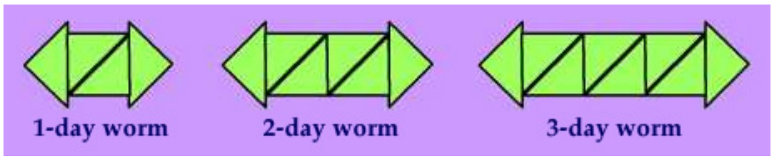
NATIONAL COUNCIL OF TEACHERS OF MATHEMATICS

Problem of the Week Teacher Packet

Growing Worms



In the land of Trianglia the worms are made of isosceles right triangles — and they grow fast! As you can see above, a worm that is 1 day old is made of 4 of these triangles. You can also see worms that are 2 days old and 3 days old. If that growth rate remains constant, how many triangles will be needed for a 4-day-old worm? a 10-day-old worm? a 63-day-old worm?

Extra: I found a worm that was made of 60 triangles. How old was it? Explain how you know.

Super Extra: Can you make a rule that uses a worm's age (in number of days) to find out how many triangles it is made of? You may express your rule with words or with numbers and symbols.

Inspired by *Navigating through Algebra in Grades 3-5*, National Council of Teachers of Mathematics.

Answer Check

After students submit their solution, they can choose to "check" their work by looking at the answer that we provide. Along with the answer itself (which never explains how to actually get the answer) we provide hints and tips for those whose answer doesn't agree with ours, as well as for those whose answer does. You might use these as prompts in the classroom to help students who are stuck and also to encourage those who are correct to improve their explanation.

A 4-day-old worm is made of 10 triangles. A 10-day-old worm is made of 22 triangles. Can you use what you learned to find the number of triangles in a 63-day-old worm?

If your answer **doesn't** match ours,

- do you realize that all the triangles count, including head and tail?
- did you try drawing a 4-day-old worm? a 5-day-old worm?
- did you make a table listing the age and number of triangles?
- did you check your arithmetic?

If any of those ideas help you, you might revise your answer, and then leave a comment that tells us what you did. If you're still stuck, leave a comment that tells us where you think you need help.

If your answer **does** match ours,

- have you clearly shown and explained the work you did?
- did you try the Extra?
- did you explain any patterns or insights you discovered while solving the problem?
- did you make any mistakes along the way? If so, how did you find and fix them?

Revise your work if you have any ideas to add. Otherwise leave us a comment that tells us how you think you did—you might answer one or more of the questions above.

Our Solutions

Method 1: Draw a Picture

I noticed that the worm grows one more square in the middle every day. Each square consists of two triangles. The head and tail stay the same. I drew a 4-day-old worm. It had 4 squares in the middle and was made of 10 triangles. I drew a 10-day-old worm. It had 6 more squares in the middle. I started at 10 and counted on six more 2s and got to 22 triangles.

I did not want to draw a 63-day-old worm, so I skip counted on a number line, using the numbers to represent the number of days. I pointed at the 10 for the 10- day-old worm. I began with 22 (triangles) on the 10th day and counted by 2s on each number on the line until I got to number 63. My skip count was at 128, so a 63-day-old worm is made of 128 triangles.

Method 2: Make a Table

We made a table listing the age of the worm in days and the number of triangles it was made of. Then we drew a 4-day-old worm and added it to the table.

Each day the worm grew by one square, or two triangles. In 6 more days the worm would grow 6 more

squares, or 12 more triangles. We added that to 10 triangles (at 4 days). !12 + 10 = 22 triangles.

Age	Triangles
1	4
2	6
3	8
4	10
5	12
6	14
7	16
8	18
9	20
10	22

After 53 more days the worm would be 63 days old. 53 days • 2 triangles per day = 106 more triangles

22 triangles (after 10 days) + 106 more triangles = 128 triangles for a 63-day-old worm.

Extra: We knew that a 4-day worm needed 10 triangles. 60 triangles is 50 more triangles than that. The worm grows by one square, or two triangles, per day. 50 triangles represent 25 days of growth because 50 ÷ 2 = 25. 4 days + 25 days = 29 days, so the worm would be 29 days old.

Super Extra: We looked at the numbers in our table and saw that you can find the number of triangles by doubling the age and adding 2 more for the head and tail. For example, to find the number of triangles for a 4-day-old worm:

(2 • 4) + 2 = 10 triangles

Method 3: Look for Patterns

From the diagram I could see that each worm contains a square of 2 triangles for each day of its age, and two extra triangles (one head and one tail). From that information I could tell that a 4-day-old worm would be made of 10 triangles:

(2 • 4) + 2 = 10 triangles

A 10-day-old worm would be made of 22 triangles:

(2 • 10) + 2 = 22 triangles

A 63-day-old worm would be made of 128 triangles:

(2 • 63) + 2 = 128 triangles

Extra: I worked backwards. First I subtracted 2 triangles for the head and tail. That left 58. I divided 58 in half, since the triangles are paired to make squares, and one pair of triangles represents one day of age.

58/2 = 29 days old

Standards

If your state has adopted the Common Core State Standards, you might find the following alignments helpful.

Grade 3: Operations & Algebraic Thinking

Solve problems involving the four operations, and identify and explain patterns in arithmetic.

Grade 4: Operations & Algebraic Thinking

Generate and analyze patterns.

Grade 5: Operations & Algebraic Thinking

Analyze patterns and relationships.

Mathematical Practices

1. Make sense of problems and persevere in solving them.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics
8. Look for and express regularity in repeated reasoning.

Teaching Suggestions

Some students may simply notice that the number of triangles increases by two each day and use a counting strategy. We call this the recursive form, as each result is based on the previous result. It is the pattern most easily discovered by students. While it is a legitimate strategy, it is limited in that, in order to find the number of triangles for any given age, we need to find that number for all the ages prior to it. Finding the number of triangles for a 63-day worm with this method becomes quite tedious.

A more sophisticated solver takes advantage of a rule based on the age of the worm, called the closed form. The advantage of this method is that the number of triangles can be directly calculated for any given age. Students who demonstrate understanding of the problem as a recursive process might be encouraged to take advantage of what they have learned in order to move toward discovery of the closed form. Consider asking questions such as: What parts of the worm change as the worm grows older? What parts remain the same no matter the age?

Students may use different methods of representing the problem and organizing their thinking, such as drawing pictures, making a table or graph, or using a spreadsheet. As they generalize any type of pattern or rule from the data in a table, they should support the continuation of that pattern by relating it to the physical model. Encourage students, for example, to explain why the total number of triangles increases by two each day based on the diagram, not simply on data in a table.

The questions in the Answer Check, above, might serve as useful prompts to help students make progress. Encourage students to use a strategy that works for them. As you can see from many methods we used in Our Solutions, above, your students might organize this problem in a number of different ways.

Sample Student Solutions – Focus on *Completeness*

In the solutions below, I've provided scores the students would have received in the **Completeness** category of our scoring rubric. My comments focus on areas in which they seem to need the most improvement.

Novice	Apprentice	Practitioner	Expert
Has written very little that explains how the answer was achieved.	Might show that their solution works without saying anything about how they figured it out. Might summarize their strategy without showing any math work to justify their answer.	Tells all of the important steps taken to solve the problem, which should include: • any relationships used. • the rationale behind each decision they made. • explaining why their answer is correct.	Adds in useful extensions and further explanation of some of the ideas involved. The additions are helpful, not just "I'll say more to get more credit."

William, age 13, Novice

4th day will be 10 triangles and The 10thday will be 22 triangles, and the 63 day will be 128

The channleage was esay. You have to find a patern and see how much the worm grew.

William's short answers are correct, but he wrote nothing to explain how he found them. It's possible that he understood the pattern. He needs to describe the pattern and how he discovered it, how it relates to the diagram, and include any calculations he used.

Christina, age 11, Apprentice

There are 10 isosceles right triangles in a four day old worm.

For every additional day, the worm increases by 2 isosceles right triangles. If a 2 day old worm is 6 triangles than a 4 day old worm would be 10 triangles.

A 10 day old worm would be 22 triangles.

A 63 day old worm would be 128 triangles.

Christina understood how the worm grows. She explained how she found the 4-day worm, but I'd like to know how she found the others. Did she draw the worms? Make a table? Use a rule? I'd ask to see any calculations.

Austin, age 11, Apprentice

1. 4 days=10 isosceles triangles
2. 10 days=22 isosceles triangles
3. 63 days=128isosceles triangles

Extra: The worm was 29 years old
Super Extra: 2n+2=number of triangles n=nuber of days

1. I did 2x4+2 and got 10 as my answer.
2. I did 10x2+2 and got 22 as my answer.
3. I did 63x2+2 and got128 as my answer.

Extra: I subtracted 2 from 60 and got 58 and divided 58 by 2 and got 29 as my answer.

Super Extra: I knew I had to use a variable for the days, multiply the days by 2, then add 2.

Austin clearly has a good mastery over the math. He generalized a good formula and used his rule inversely to solve the Extra. I'd ask for his reasons for multiplying the days by 2, and then adding 2. What do the two 2s in his formula represent in terms of the diagram and the growth of the worm? I'd remind him about correct use of the equals symbol, a Clarity issue.

Robin, age 10, Apprentice

For a four day worm you will need ten triangles, for a ten day old worm you wiil need 22 triangles and for 63 day old worm you will need 128 triangles.

Days Triangles
1 > < 4
2 > < 6
3 > < 8
4 > < 10
5 > < 12
6 > < 14
7 > < 16
8 > < 18
9 > < 20
10> < 22
I made a chart up to ten and then I noticed that if you add the number of days to itself plus two you get the answer. So to figure out 63 days I did
63 x 2 + 2 = 128.

Extra- I took 60 and subtracted two so it was 58. After that I divided 58 by two and that made 29.

Robin did a good job of including a table and finding a formula. She applied the formula accurately to the 63-day worm. I'd like to know how she carried out the table beyond the 3-day worm in the diagram. If from the +2 pattern in the table, how does the diagram support that? She carried out her inverse steps correctly in the Extra. I'd ask why she did those steps and why in that order.

Masha, age 10, Practitioner

A 10 day old worm is 22 triangles. A 63 day worm is 128 triangles. A 4 day Old worm is 10 triangles. The rule I have is: multiply the day # by 2, then add 2. The 60 triangle worm was 28 days old.

I figured out that there were the # of squares in the worm as the # of days, And that each square has 2 triangles. Using that, I multiplied each of the day old numbers and then added two triangles, then, because there were two at the ends of each one. Knowing that I did that, I knew what stradegy I used. To figure out the 60 triangle one, I just did the stradegy backwards: divide by 2, subtract 2.

Masha makes a strong connection between her rule and the physical model. I'd ask her to clean up some Clarity issues, e.g. # for the word "number." I'd ask her to check the 28-day worm with her rule. This will help her understand that, when applying inverse operations (great strategy!) one needs to reverse the order as well (subtract first and then divide by 2).

Daniel, age 12, Practitioner

In four days the worm will have 10 triangles. In 10 days it will have 22 triangles. And in 63 days the worm will have 128 triangles.

The first thing I did was look at the worms and counted how many triangle They had. Then I tried to figure out a pattern. I noticed that as the days went up by one, each worm got 2 more triangles. On the third day the worm had 8 triangles, so I added two and got 10 triangles for the 4th day. Then I had to figure out how many triangles there would be in 10 days. So I counted up by two each time. Day five had 12 triangles, 6 had 14, 7 had 16, 8 had 18, 9 had 20, so 10 would have 22.
Now I had to figure out how many triangle there would be in 63 days. I wasn't going to count like I did for 10 all the way up to 63, so I had to figure out a different way. I was going to go up by tens. 10 had 22 so 20 would have 42. So if I went by that by the time I got to 60 it would have 122 triangles. I figured this out because 10 had 22 and that was 2 more than 10 *2. So 20*2=40 then I added 2 to 40 and got 42. So now that I new that 60 had 122 triangles, I could count up like I did with 10. So 61 has 124, 62 has 126, so 63 would have 128 triangles. I am sure I am correct because, the answer is 2 more than double the age.

Daniel described in detail his method of gradually working his way up by twos, always relating his numbers to the worms. His process of constructing the 63-day worm demonstrated good number sense. His final statement generalizing a rule would count as a reflection, as he uses a different method to check his calculations. Daniel is ready to tackle the Extra.

Daniel, age 7, Practitioner

A 4 day old worm will need 10 triangles
a 10 day old worm will need 22 triangles
a 63 day old worm will need 128 triangles

Extra: A worm made of 60 triangles will be 29 days old.

SuperExtra: If the number of days old = x, then the number of triangles needed will be 2x+2.

I noticed that the number of triangles needed for each worm followed an arithmetic sequence with a difference of +2.

I also noticed that if you removed the 2 end triangles, the number of triangles needed was twice the number of days which gave me the formula 2x+2 when x was the number of days old.

Therefore a 4 day old worm was 2(4)+2 = 10

a 10day old worm was 2(10)+2 = 22
a 63 day old worm was 2(63)+2 = 128

Extra: To find how old a worm was that was made of 60 triangles, I started with the formula 2x+2=60. I need to get x by itself so I used inverse operations. First I subtracted 2 from both sides so I had 2x=58. Then I divided both sides by 2 which gave me 29=x.

Daniel's thinking and his communication skills are excellent. He justifies his formula with details from the diagram. He clearly understands inverse operations I would ask him to explain how his inverse procedure relates to the physical model. Why does it make sense in terms of the diagram to subtract 2 before dividing?

Emily, age 10, Expert

The answer is a 4 day old worm has 10 triangles, a 10 day old worm has 22 triangles, and a 63 day old worm has 128 triangles. The answer to the extra is a worm that has 60 triangles is 29 days old.

I know that the pattern adds two triangles each day. So I will multiply the day by two, then add two because of the triangles on the outside of the worm. Also, all of my answers will be even because the first worm started out even and adds even numbers each day.

1) 4x2=8+2=10 2) 10x2=20+2=22 3) 63x2=126+2=128

So the answer is the 4 day old worm is 10 triangles, the 10 day old worm is 22 triangles and the 63 day old worm is 128 triangles.

I checked my work by subtracting 2 then dividing it in 2.

1) 10-2=8 8/2 =4 2) 22-2=20 20/2=10 3) 128-2=126 126/2=63

EXTRA: Using the first method in reverse, I did 60-2=58. Then I did 58/2. So my answer is the worm is 29 days old. I checked my work by doing 29x2=58+2=60.

*Emily explained her rule and how it related to the diagram. She included her calculations for the main problem and the Extra. She provided exceptional insight in the problem by explaining why the numbers of triangles need to be even. Her Extra explanation could be improved by connecting it to the diagram. I would model for her a more accurate way to notate her calculations, e.g., 4 * 2 + 2 = 10*

Scoring Rubric

A **problem-specific rubric** can be found linked from the problem to help in assessing student solutions. We consider each category separately when evaluating the students' work, thereby providing more focused information regarding the strengths and weaknesses in the work.

We hope these packets are useful in helping you make the most of the Math Fundamentals Problems of the Week. Please let me know if you have ideas for making them more useful.

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