NATIONAL COUNCIL OF TEACHERS OF MATHEMATICS

Problem of the Week Teacher Packet

Ostrich Llama Count

Raul and Esteban just started working at their uncle's farm on the weekends. Their first task was to count the ostriches and llamas. When they reported to their uncle,

Raul said, "I counted 47 heads."

Esteban added, "I counted 122 legs."

"How many are ostriches? How many are llamas?" asked their uncle.

"It's getting dark and I promised your mother I'd get you home for dinner. There's no time to count again. You'll have to figure out how many ostriches and how many llamas there are from that information when you get home. Can you give me a call after dinner and let me know your answer?"

How did Raul and Esteban figure out how many ostriches and how many llamas there were?


ostrich


llama

Answer Check

After students submit their solution, they can choose to "check" their work by looking at the answer that we provide. Along with the answer itself (which never explains how to actually get the answer) we provide hints and tips for those whose answer doesn't agree with ours, as well as for those whose answer does. You might use these as prompts in the classroom to help students who are stuck and also to encourage those who are correct to improve their explanation.

There are 33 ostriches and 14 llamas.

If your answer **doesn't** match ours,

- did you remember how many legs a llama has and how many legs an ostrich has?
- might a table help you keep track of things?
- did you consider thinking algebraically?

If you used guess and check, did you tell . . .

- what numbers you tried?
- how you knew whether they worked or not?
- how you decided what to try next?

If any of those ideas help you, you might revise your answer, and then leave a comment that tells us what you did. If you're still stuck, leave a comment that tells us where you think you need help.

If your answer **does** match ours,

- are you confident that you could solve another problem like this successfully?
- is your explanation clear and complete?
- did you make any mistakes along the way? If so, how did you find them?
- what hints would you give another student trying to solve this problem?

Revise your work if you have any ideas to add. Otherwise leave us a comment that tells us how you think you did—you might answer one or more of the questions above.

Our Solutions

Method 1: Guess and Check: Make a chart

I created 4 columns showing # of ostriches, # of llamas, #of legs for ostriches and # of legs for llamas. Then, through trial and error, I put numbers in these columns so that the animals numbered 47 and the legs equaled "whatever." Here are the number columns I used:

Ostriches	Llamas	Ostrich legs	Llama legs
20	27	40	108
27	20	54	80
30	17	60	68
31	16	62	64
32	15	64	60
33	14	66	56

When the proper combination of numbers equaled 47 animals and 122 legs I knew I had the correct combination of animals.

Method 2: More sophisticated Guess and Check: Make a more detailed chart

I found out how to get my answer by matching two numbers, like 14 llamas and 33 ostriches, to get 47 heads and 122 legs. I used this information stated in the problem:

- There are 47 heads (so 47 animals)
- There are 122 legs altogether
- Ostriches have 2 legs and llamas have 4 legs

I needed to figure out how many animals there were and to match those numbers with the correct number of legs and heads that there were. I used a guess and check method. To make this easier to visualize, I made a chart.

# of llamas	# of legs	# of ostriches	# of legs	total animals heads	total legs	does it match? 122 legs, 47 heads?
15	60	32	64	47	124	no
13	52	34	68	47	120	no
14	56	33	66	47	122	yes

I guessed a number of llamas, multiplied it by 4 because each llama has 4 legs. The reason I guessed 15 llamas as a number to start with was just a lucky guess, it turned out to be a pretty close number to the actual answer. I subtracted the guessed number of llamas from 47 (heads) to find the number I would use for ostriches. I multiplied that number by 2 because ostriches have 2 legs.

My next step was to add the number of legs together to see if it would equal 122. We knew the number of animals because we had already done the subtraction step to get an amount equal to 47 heads. This is how Raul and Esteban might have figured out how many ostriches and llamas there were.

Method 3: Logical Reasoning

There were 47 heads which means there were 47 animals. Ostriches have 2 legs and llamas have 4 legs. Both ostriches and llamas have at least two legs. So, I multiplied 47 heads x 2 legs = 94 legs.

Since Esteban counted a total of 122 legs, I subtracted the 94 legs from 122 and the difference was 28 legs. Now I have 28 legs left to attach to heads. These 28 legs must be attached by 2's to make 4 legged llamas. So,I divided the remaining 28 legs by 2 which equals 14 pairs, and so there were 14 animals with 4 legs (llamas).

I had found out there were 14 llamas. Since llamas and ostriches have 1 head each, I subtracted the 14 llamas from the 47 heads and found out there were 33 heads with 2 legs each (ostriches).

I checked my work by multiplying 2 legs x 33 ostriches which equals 66 legs. Next I multiplied 4 legs x 14 llamas which equals 56 legs. Then I added 66 legs + 56 legs and it equaled 122 legs.

Method 4: Algebraic Reasoning

Since there were 47 heads total and each animal has one head, the total number of heads would be:
number of ostrich heads + number of llama heads = 47

We also know that ostriches have two legs and llamas have four, so:
two times (number of ostrich heads) + four times (number of llama heads) = 122

How do we use these two number sentences to find out the number of each animal? We can think of the first one like this:
number of ostrich heads = 47 - (number of llama heads)

And, now I can use that with the second number sentence:
two times [47 - (number of llama heads)] + four times (number of llama heads) = 122

If I use the distributive property,
94 - [two times the number of llama heads] + [four times the number of llama heads] = 122
94 + [two times the number of llama heads] = 122
[two times the number of llama heads] = 28
the number of llama heads equals 14

If I use "14" in my first number sentence instead of the "number of llama heads," then I would have
two times (number of ostrich heads) + four times (14) = 122
two times (number of ostrich heads) + 56 = 122
two times (number of ostrich heads) = 66
the number of ostrich heads equals 33

Method 5: Algebra

There are 33 ostriches and 14 llamas.

Let x = number of ostriches
y = number of llamas
(I would like to choose "o" and "l" but didn't since they look too much like numbers)

Since there were 47 heads total and each animal has one head (we assume...) the sum can be represented as ostriches plus llamas:

$$x + y = 47$$

Ostriches have two legs and llamas have four, so we can multiply the variables by 2 and 4, respectively, to represent the total number of legs:

$$2x + 4y = 122$$

Now we can solve the system of two variables with two equations, and there are several ways to do this. I've chosen substitution

$$x + y = 47$$
$$y = 47 - x$$

$$2x + 4y = 122$$
$$2x + 4(47 - x) = 122$$
$$2x + 188 - 4x = 122$$
$$-2x = -66$$
$$x = 33$$

$$y = 47 - x$$
$$y = 47 - 33$$
$$y = 14$$

Standards

If your state has adopted the Common Core State Standards, you might find the following alignments helpful.

Grade 6: Ratios & Proportional Relationships

Use ratio and rate reasoning to solve real-world and mathematical problems e.g., by reasoning about tables of equivalent ratios, tape diagrams, double number line diagrams, or equations.

Grade 7: Ratios & Proportional Relationships

Analyze proportional relationships and use them to solve real-world and mathematical problems.

Grade 7: Expressions & Equations

Solve real-life and mathematical problems using numerical and algebraic expressions and equations.

Grade 8: Expressions & Equations

Solve real-world and mathematical problems leading to two linear equations in two variables.

Mathematical Practices

1. Make sense of problems and persevere in solving them.
3. Construct viable arguments and critique the reasoning of others.

Teaching Suggestions

Resist the urge to give direct instructions on a specific approach. Ask students to paraphrase the problem to check on their understanding before they begin working on it. Ask questions that help them understand the language of the problem, visualize it, and discover patterns. Good questions help students clarify their thinking and give you useful information as well. The questions in the Answer Check, above, might serve as good prompts to help students make progress.

A variety of interesting strategies can be applied to ***Ostrich Llama Count***. Students can use increasingly sophisticated guess and check strategies, logical reasoning strategies, or algebraic strategies. In each case, there are opportunities for students to notice patterns and relationships, for example:

- how many ostriches equal one llama (when you're counting legs)?
- if I replace one llama with one ostrich, what happens to the number of legs?

This problem would be a fun one for having groups of students try to generate as many strategies as possible, and compare and contrast the relationships utilized by each strategy. For example, a student who used a table and a student who used logical reasoning might notice that they both realized that replacing one ostrich with a llama increases the number of legs by two.

We considered listing a method that uses a "combination chart," a tool but decided instead to suggest it in this section. In using "Comparing Quantities" from the *Math in Context* curriculum, we have become familiar with their combination charts. The idea is that if you have two items that you will purchase or count, you can make a chart to see what their combined values would be. In the chart below you can see that the columns are numbered starting at 0 and indicate the number of ostriches. The rows are numbered also starting at 0 and indicate the number of llamas. To fill the chart, we thought about the numbers of legs in each case. Here's a simple case where we've thought on the number of legs the first few combinations of llamas and ostriches might have.

	5	20	22	24	26	28
	4	16	18	20	22	24
# of llamas 4 legs each	**3**	12	14	16	18	20
	2	8	10	12	14	16
	1	4	6	8	10	12
	0	0	2	4	6	8
		0	**1**	**2**	**3**	**4**
		# of ostriches -- 2 legs each				

If we have 2 ostriches and 1 llamas, we can read from the chart that combined there are 8 legs. If we have 3 ostriches and 4 llamas, we can read from the chart that combined there are 22 legs.

This method works well with small numbers of items. We knew that we would have 47 animals total since that was the number of heads counted. Once we thought of using a spreadsheet, using the combination method seemed manageable. Once the rows and columns are set up and the first few numbers are added to represent the number of legs, the “autofill” feature comes in very handy!

Combined Number of Legs

	14	56	58	60	62	64	66	68	70	72	74	76	78	80	82	84	86	88	90	92	94	96	98	100	102	104	106	108	110	112	114	116	118	120	122
	13	52	54	56	58	60	62	64	66	68	70	72	74	76	78	80	82	84	86	88	90	92	94	96	98	100	102	104	106	108	110	112	114	116	118
	12	48	50	52	54	56	58	60	62	64	66	68	70	72	74	76	78	80	82	84	86	88	90	92	94	96	98	100	102	104	106	108	110	112	114
	11	44	46	48	50	52	54	56	58	60	62	64	66	68	70	72	74	76	78	80	82	84	86	88	90	92	94	96	98	100	102	104	106	108	110
	10	40	42	44	46	48	50	52	54	56	58	60	62	64	66	68	70	72	74	76	78	80	82	84	86	88	90	92	94	96	98	100	102	104	106
	9	36	38	40	42	44	46	48	50	52	54	56	58	60	62	64	66	68	70	72	74	76	78	80	82	84	86	88	90	92	94	96	98	100	102
	8	32	34	36	38	40	42	44	46	48	50	52	54	56	58	60	62	64	66	68	70	72	74	76	78	80	82	84	86	88	90	92	94	96	98
# of llamas	**7**	28	30	32	34	36	38	40	42	44	46	48	50	52	54	56	58	60	62	64	66	68	70	72	74	76	78	80	82	84	86	88	90	92	94
4 legs each	**6**	24	26	28	30	32	34	36	38	40	42	44	46	48	50	52	54	56	58	60	62	64	66	68	70	72	74	76	78	80	82	84	86	88	90
	5	20	22	24	26	28	30	32	34	36	38	40	42	44	46	48	50	52	54	56	58	60	62	64	66	68	70	72	74	76	78	80	82	84	86
	4	16	18	20	22	24	26	28	30	32	34	36	38	40	42	44	46	48	50	52	54	56	58	60	62	64	66	68	70	72	74	76	78	80	82
	3	12	14	16	18	20	22	24	26	28	30	32	34	36	38	40	42	44	46	48	50	52	54	56	58	60	62	64	66	68	70	72	74	76	78
	2	8	10	12	14	16	18	20	22	24	26	28	30	32	34	36	38	40	42	44	46	48	50	52	54	56	58	60	62	64	66	68	70	72	74
	1	4	6	8	10	12	14	16	18	20	22	24	26	28	30	32	34	36	38	40	42	44	46	48	50	52	54	56	58	60	62	64	66	68	70
	0	0	2	4	6	8	10	12	14	16	18	20	22	24	26	28	30	32	34	36	38	40	42	44	46	48	50	52	54	56	58	60	62	64	66
		0	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**	**13**	**14**	**15**	**16**	**17**	**18**	**19**	**20**	**21**	**22**	**23**	**24**	**25**	**26**	**27**	**28**	**29**	**30**	**31**	**32**	**33**

# of ostriches -- 2 legs each

Sample Student Solutions – Focus on *Strategy*

In the solutions below, I’ve provided scores the students would have received in the **Strategy** category of our scoring rubric. My comments focus on areas in which they seem to need the most improvement.

Novice	Apprentice	Practitioner	Expert
Has no ideas that will lead them toward a successful solution or shows no evidence of strategy.	Uses a strategy that uses luck instead of skill, or doesn’t provide enough detail to determine whether it was luck or skill.	Uses a strategy that relies on skill, not luck, which might include: • thorough noticing and wondering • guess and check • make a table • logical reasoning • algebraic reasoning	Might use a formal algebraic method or discuss several different ways to generate possible solutions.

David, age 13, Novice

Raul and Esteban used this expression to figure out the problem. 4L+2O=122 L=Llamas=14 O=Ostriches=33 There are 14 llamas and 33 ostriches.

The stratagies I used were guess and check and use an algebraic expression. I the expression 4L+2O=122 to help me figure out this problem.I used this expression because if I didn't, it wouldn't be very easy to use the guess and check meothod. (My teacher healped me figure out the expression)I then used the guess and check method to find out what L and O equals in my expression. The first time I tried guessing I used the numbers 11,and 11 because they were 2#'s that were the same and easy to work with but 11*11=121. So then I tried 10 and 12 and it didn't work either just as 9 and 13 didn't work. If you keep

David understands how to calculate the number of legs based on the number of ostrich heads and of llama heads, and even expressed his understanding algebraically. Even though he does get the correct answer, the strategy he describes would not work in general. David tests that the using the pattern I used you will eventually get to 14 and 33.14 *33=122 so the #'s work. Then I just pluged those 2 #'s into my expression. 4*14+ 33*2=122.It took me a while to figure out what L and O equals but I just keept guessing and checking untill I got 122 for the answer.

number of legs equals 122, but he doesn't describe testing that the number of heads equals 47, so he could just as easily have gotten 30 llamas and 2 ostriches.

Also, he didn't have a way to organize his guesses; it seems that he might have tried every pair of integers.

I'd ask David to think about what he knows about the number of heads, and if that would help him find a way to make a different pattern, so that it didn't take him quite so long to find his answer.

Panthers (team), age 10, Novice

The answer is they divided 47 by 2, finding out that it wasn't even, then finding out the best way to sort out the number 47.

First, we looked at the number 47. Since 47 was odd, we knew it wouldn't come out divided evenly, but we divided it anyway to get started up. After that, we divided 47 as best as we could in whole numbers, getting 23 and 24. Then, we checked the laamas, because only 1 animal for the checking was necessary, and looked at 24 first. Then, we divided 24 into 122 to see if it divided into there by an equal whole number without decimals. 122 divided by 24 = 5. ongoing. That meant that there MUST have been 23 llamas. That mean that there were 24 ostriches.

The Panthers figured out that the number of heads had to be 47, and that they had to figure something out about the number of legs. It seems from their strategy that they assumed the number of animals had to be as close to equal as possible, and that they merely had to figure out which type of animal went with the 122 legs.

I would ask the Panthers if they could tell me how many legs 24 ostriches and 23 llamas have all together, and how many legs the animals are supposed to have all together.

I might also ask them if just checking 1 animal was enough, by asking them what would have happened if they had checked ostriches instead.

Maureen and Rachel, age 13, Apprentice

The way that the children found the number of each kind of animal was by taking the number 47 (number of animals all together) and subtracting the number 2 (the nuber of ostrich legs) as many times as it takes to get to a number divisible by 4(the number of llama legs). Then they took the legs of the ostrich and multipled them by 2 and added that by the number of llama legs multiplyed by 4 which equals 122 . There are 33 ostriches and 14 llamas.

Maureen and Rachel, while they seem to have gotten the correct answer, could not have gotten that answer by the strategy they describe. They do seem to understand the quantities in *the problem, and how to calculate the number of legs.*

I would ask them questions that help them clarify their strategy, for example, "I'm a little confused by your strategy. No matter how many times I subtract 2 from 47, I never get a number divisible by 4. Could you show me some of the numbers you tried along the way, and how you figured out when you had a number that worked?"

Chris, age 13, Apprentice

Raul and Esteban found their answer by plugging in different numbers for the number of ostriches and the number of llamas, and adding them to see if they equal the amount of heads and legs.

First, I saw that the amount of ostriches and llamas had to equal to the amount of legs and heads. Llamas have four legs, and ostriches have two legs. I began plugging in amounts of llamas and ostriches that would equalm to 47 heads. Once I found numbers that equaled to that, I tried to find an amount of legs that would equal to 122 legs. After plugging in many numbers, I found that if there were 14 llamas and 33 ostriches, I found that that equaled to 47 heads and 122 legs. There were 33 ostriches, and 14 llamas.

Chris seems to have an effective guess-and-check strategy, in that he probably knows how to make sure his answer fits the requirements. It seems that he plugs in numbers of ostriches and llamas that total to 47 animals, and checks that the number of legs is correct. However, his description is unclear because he doesn't show his guesses or how he checks them. For example, it almost sounds like he satisfies the heads = 47 condition separately from the legs = 122 condition. Chris also doesn't show if he has a systematic way of keeping track of his guesses.

I would ask Chris if he could show us what numbers he tried, and how he checked to see if the number of heads and the number of legs were correct. I would suggest that he try to use a very organized method, such as a table or a lined-up list, to show us his guesses and calculations.

Koji, age 9, Practitioner

There are 33 ostriches and 14 llamas.

First I drew a picture of 122 legs and then grouped by 4. I got 30 groups of 4 and 1 pair. That didn't make 47. So I grouped in 4, 2,4, 2 pattern. Then I got 20 llamas and 21 ostriches. They still didn't make 47. So I took away 1 llama that equals 2 ostriches. Then I kept doing the same thing until I got 47 in total number of the animal.

Koji employed a visual guess and check strategy. He kept the number of legs at 122, and grouped them into llamas and ostriches, changing his grouping pattern until he got 47 animals. One cool aspect of his strategy is that he figured out a way to adjust it… once he got 21 ostriches and 20 llamas, he knew he needed more animals, so he broke certain llamas apart into 2 ostriches until he got 47 animals.

I would want to push Koji to abstract or generalize his strategy, by asking him, "Koji, I have some questions about how you might use this strategy on a similar problem. When you took away 1 llama and replaced it with 2 ostriches, how much did the total number of animals change by? Could you make a prediction about how many llamas you would need to replace?"

Jackie, age 13, Practitioner

There were 14 llamas and 33 ostriches. They figured this out by trying all possible combinations. Raul and Esteban figured it out from their information they got earlier. 47 heads means 47 animals in total and there was 122 legs.

The way to figure this out is by trying all possible combinations.

Knowing that a llama has 4 legs and a ostrich has 2 legs, the calculation I did was this: o = ostrich l = llama

47 o, 0 l = 47x2(legs)= 94 legs; 94 legs is less than 122 legs so its not correct.
46 o, 1 l = 46x2=92, 1x4= 4; 92+4=96 legs
45 o, 2 l = 45x2=90, 2x4= 8; 90+8=98 legs
44 o, 3 l = 44x2=88, 3x4= 12; 88+12=100 legs
43 o, 4 l = 43x2=86, 4x4= 16; 86+16=102 legs
42 o, 5 l = 42x2=84, 5x4= 20; 84+20=104 legs
41 o, 6 l = 41x2=82, 6x4= 24; 82x24=106 legs
40 o, 7 l = 40x2=80, 7x4= 28; 80+28=108 legs
39 o, 8 l = 39x2=78, 8x4= 32; 78+32=110 legs
38 o, 9 l = 38x2=76, 9x4= 36; 76+36=112 legs
37 o, 10 l = 37x2=74, 10x4= 40; 74+40= 114 legs
36 o, 11 l = 36x2=72, 11x4= 44; 72+44=116 legs
35 o, 12 l = 35x2=70, 12x4= 48; 70+48=118 legs
34 o, 13 l = 34x2=68, 13x4= 52; 68+52=120 legs
33 o, 14 l = 33x2=66, 14x4= 56; 66+ 56=122 legs

Jackie's guess and check strategy and explanation are very thorough and clear. Since she has such a well-organized table, there are a lot of opportunities to help her notice patterns.

I would ask her, "From your table, can you figure out what happens to the number of legs each time you add in another llama? Could you use that to predict how many llamas you need to add?"

33 ostriches and 14 llamas is the answer. 33 ostriches + 14 llamas equals 47 animals and since each animal has one head, there are 47 heads in all just like Raul said. Since ostriches have 2 legs and llamas 4, 33 ostriches have a total of 66 legs and 14 llamas have a total of 56 legs. 66 legs plus 56 legs equals 122 legs just like Esteban said.

If you continue on the chart, you will not get 122 legs from 47 animals. You can also reverse the chart, meaning you can do llamas first and then ostriches.

Cori, age 14, Expert

There are 14 llamas and 33 ostriches on the farm.

First set the equation up with a x and y value let statement:

Let x=ostriches

y=llamas

```
  x + y  = 47
 2x + 4y = 122

  x + y  = 47
     -y    -y
      x  = 47 - y

2(47 - y) + 4y = 122
  94 - 2y + 4y = 122
      -2y + 4y = 122 - 94
            2y = 28
            /2   /2

             y = 14 llamas

         x + y = 47
        x + 14 = 47
             x = 33
```

I consider Cori to have an expert strategy, since she used algebra to set up and solve the problem correctly. However, in communicating about the problem, she doesn't show that she checked to make sure her solution makes sense. She also doesn't provide any rationale for the two equations or why she should be solving a system of equations.

I would ask her, "Cori, can you explain to me how you knew to make two equations? I'm also curious about the equation 2x + 4y = 122. What does that represent in the problem?"

Bonus Section

The solutions in this section all employ logical reasoning strategies that demonstrate a firm grasp of the quantities in the problem and how they relate to one another. Understanding those relationships is an important step in moving towards being able to set up and solve equations, and to understand functions and their graphical, symbolic, and numerical representations.

Genia, age 11

Say that we have one ostrich. An ostrich has 2 legs. We take 2 legs from 122 legs. We have 120 legs left. That means we have 30 llamas because llamas have 4 legs and you divide 120 by 4. That's 31 heads.

Say we have... Wait! Let's make a chart

ostrich|llama|heads
1 | 30 | 31
2 | 29.5| impossible
3 | 29 | 32
4 | 28.5| impossible
5 | 28 | 33
6 | 27.5| impossible
7 | 27 | 34...

Genia applied a very systematic guess-and-check strategy, and she also noticed a pattern, essentially that the number of llamas and the number of heads is a function of [changes predictably based on changing] the number of ostriches.

Subtract the quantity of the # of ostriches multiplied by two from the # of legs, then divide the result by 4, the # of llama legs, to get the # of llamas. Add that to the # of ostriches to get the # of heads. s = # of ostriches, L = # of llamas, 122 = # of legs, 2 = # of ostrich legs, and 4 = # of llama legs. So we have a formula:

[122 - (2s)] / 4 = L, s + L = # of heads

For example: [122 - (5 * 2)] / 4 = 28 llamas, 5 + 28 = 33: # of heads.

I've already noticed that there can't be an even number of ostriches because you will have half a llama! That's gross! And, as the number of ostriches goes up by two, the number of llamas goes down by 1 and the number of heads goes up by 1. Let's make a new chart

ostrich	llama	heads	
1	30	31	
3	29	32	See, I've eliminated the even numbers of ostriches
5	28	33	because of the thing I explained above with halfs
7	27	34	of llamas.
9	26	35	You know, I think we're far away, considering
21	20	41	we're going up by one. I'll eliminate 11,13,15,17,
23	19	42	and 19 to make this list a little shorter.
25	18	43	
27	17	44	
29	16	45	
31	15	46	
33	14	47	There it is!

If you put a name to my proccess, it's guess and check, even though I found a pattern in my guesses.

There were 33 ostriches and 14 llamas.

To check the answer, multiply 33 by 2, so you get the number of ostrich legs. Then multiply 14 by 4 to get how many llama legs there are. Then add the sums to get how many legs there are. You should get 122, or else you did something wrong.
33 * 2 = 66
14 * 4 = 56
56 + 66 = 122 My answer is right.

My questions for Genia might be, "I wonder if your pattern could help you predict how many ostriches would be needed to make 47 heads, before filling out the whole table"

Kristi, age 12

There are 14 llamas, and 33 ostriches on their uncle's farm.

When I first read the problem, it seemed pretty easy to me. I knew that both ostriches and llamas had at least two legs, so since there were 47 heads, I multiplied 47 by 2 and got 94 legs.

47 • 2 = 94 legs used

Then I subtracted that from 122, because I knew that 94 legs had to have been accounted for. I got 28.

122 - 94 = 28 legs left

I knew that llamas had four legs, and every animal aready had two, so to make some of them llamas, I divided 28 by two to get 14, the amount of llamas, because that was the amount of extra pairs of legs.

28 / 2 = 14
14 llamas

To find out how many ostriches there were, I subtracted 14 from 47 and got 33.

I noticed a lot of cool things about Kristi's method. One thing I noticed is that Kristi's method could also be represented algebraically, in a way that illustrates the reasoning behind solving a system of equations by linear combination. Here's my take:

let # of ostriches = s, and # of llamas = a (since o and l look like the numbers 0 and 1)

s + a = 47 (heads: this isn't explicit, but she is clearly aware that the number of

33 ostriches

I think that my answer makes sense because if you do the math going backwards, you get 47 heads and 122 legs too.

Also, if you just look at the numbers, it fairly makes sense, the numbers aren't extremely odd and strange, they just kind of fit. I learned that you can work backwards if you are given a problem like this where you have to work backwards.

47 • 2 = 94   122- 94 = 28 / 2 = 14 llamas

47 - 14 = 33 ostriches

ostriches + the number of llamas is the total # of heads)

2s + 2a = 94 ("I knew that both ostriches and llamas had at least two legs, so since there were 47 heads, I multiplied 47 by 2 and got 94 legs." She is thinking of the 94, but again, she's aware that the 94 is twice the number of ostriches plus twice the number of llamas)

4s + 2a = 122 (total # of legs)

4s + 2a - (2s + 2a) = 122 - 94 ("Then I subtracted that from 122, because I knew that 94 legs had to have been accounted for." When she subtracts 94 from 122, while she's not explicit about what's going on, she clearly has in mind that 122 represents 4 legs/llama plus 2 legs/ostrich, and she's taking away 2legs/llama and 2legs/ostrich)

2s = 28 ("I knew that llamas had four legs, and every animal already had two, so to make some of them llamas, I divided 28 by two to get 14, the amount of llamas, because that was the amount of extra pairs of legs." Because she was aware in the previous step that the 28 unaccounted for legs represented 2 legs/llama, she knew to divide the total number of unaccounted for legs by 2 to get total number of llamas)

What I love about this is that students are rarely encouraged to think informally about "messing around" with one equation in a system of equations, and then adding, subtracting, or otherwise substituting the equation, based either on the equation or the meaning of the problem, but in fact it's *a method that works really well.*

Note that I wouldn't encourage Kristi to try to formalize her reasoning this way. It's an enormous leap. What I would try to do is encourage her to generalize the method to other problems, or articulate it like she's helping another student approach another problem in a similar way." So what I might actually say to Kristi is "I wonder if your method would help you solve this problem: I have some dimes and some quarters. I have 11 coins, and their total value is $2.00. How many are dimes?

Valerie, age 13

There are 14 llamas and 33 ostriches.

First I added up the number of 1 llama's leg and 1 ostrich's leg(6 legs) because 122 is close to 120, which 6 evenly goes into. I found that 20 pairs of llamas and ostriches that would go into 120.Right away I added 1 ostrich to get 122 because that would be 2 more legs(which equals the right amount of legs). There werent enough heads, so I knew that I had to take away some llamas to get the right number of heads and 122 legs(2 ostriches for every one llama). That gave me more ostrich heads because I subtracted any number of llamas and then added 2 ostriches for each llama. I made sure that the number of heads would equal 47 and the number of legs would equal 122.I found that there were 14 llamas(56legs) and 33 ostriches(66 legs). The strategy that the boys used was guess and check.

Valerie sees the relationship that 2 ostriches can replace 1 llama, and still keep the number of legs constant, which means she can use a strategy of adjusting an initial guess. She also sees that replacing llamas with ostriches increases the number of heads.

My question for Valerie would be, "When you replace a llama with two ostriches, how does the total number of heads change (a table might help you find out)? I wonder how you could use this information to predict how many llamas you need to replace"

Rebecca, age 13

In the paddock there are 14 Lama and 33 ostriches

I worked out that if there were 47 heads there had to be 47 animal because no animal has two heads.

then i wrote down all the different combinations that add up to 47.

i pick one of the combinations: 5 lamas 42 ostriches

i multiplied the lamas by 4 and the ostriches by 2 (number of legs)
5*4 + 42*2 = 104

i then repeat the step with the next combination 6 lamas 41 ostriches

i multiplied the lamas by 4 and the ostriches by 2 (number of legs)

Rebecca's strategy is similar to one that Dr. Math described as a more powerful version of guess and check. Rebecca started with two guesses, but she generalized from her guesses to relate a change in llamas to a change in # of legs (make # of legs a function of # of llamas). She then took her observation

6*4 + 41*2 = 106

i workede out that the answer went up by twos each time.
my answer need two go up 8 times more so...

i did 14*4 +33*2 = 122

and i got my answer

one step further, to figure out how many llamas are needed to make 122 legs.

My feedback for Rebecca: "I wonder how you figured out that your answer needed to go up 8 more times, and how you knew, if your answer needed to go up 8 more times, how many llamas and ostriches to test. I also wonder how your method could be applied to this problem I found: I have some dimes and some quarters. I have 11 coins, and their total value is $2.00. How many are dimes and how many are quarters?"

PS -- I'd say this method is roughly equivalent to the substitution method, the same way the two problems above are roughly equivalent to linear combination. Which, again, may or may not mean that there's a clear bridge from the intuitive to algebraic version.

Andrei, age 13

There are 33 ostriches and 14 llamas.

I noted with O the number of ostriches and with L the number of llamas. Knowing that the ostriches and the llamas have each only one head, I see that the total number of heads is the number of animals. So, I have:

$O + L = 47$

Now, I look at the legs. An ostrich has two legs and a llama has four legs. So, the total number of legs is two times the number of ostriches plus four times the number of llamas:

$2O + 4L = 122$

Now, I have two equations and two unknowns:

$O + L = 47$
$2O + 4L = 122$

In the following, I shall keep the two equations together, to see all the time the system.

In the second equation, I divide all terms by 2:

$O + L = 47$
$O + 2L = 61$

Now, I subtract the first equation from the last one, and I keep the first unchanged:

$O + L = 47$
$O + 2L - O - L = 61 - 47$

or:

I noticed that Andrei's algebraic and visual solutions are basically identical, but that the visual solution seems much more intuitive, much easier for a younger student who does not yet know the formal algebra to use.

I wonder if some of the students above, like Kristy, could use Andrei's method to bridge from their intuitive methods to an algebraic method.

In my feedback for Andrei, I might ask: "I wonder what the pros and cons are of the algebraic vs. the visual method, or are they basically the same."

O + L = 47
L = 14

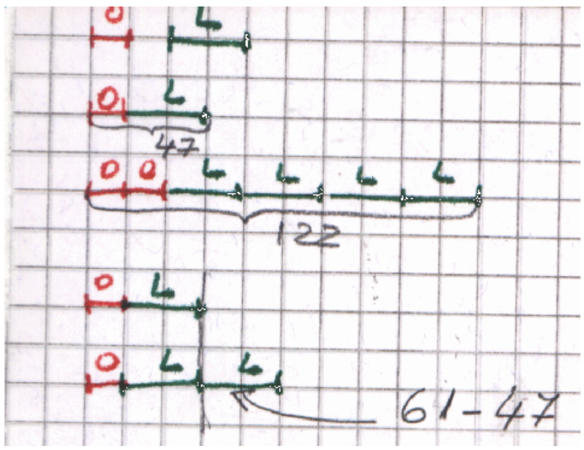
Now, I obtained L. Here I calculate O from the first equation:

O = 33
L = 14

Now, I verify my answer:

O + L = 47
2O + 4L = 122

This was using algebra. Some time before, before knowing about algebra, I solved this type of problem using arithmetic: the easiest way, as I remember, is to associate lengths of segments to the number of ostriches and to the number of llamas (see picture):



If the red segment is associated with the number of ostriches, and the green one with the number of llamas, adding their heads I obtain 47. Then, each ostrich has 2 legs, and each llama 4 – I obtain 122 legs. I see that the last segment, is composed by 2 red units and 4 green, so that I could divide its length in two using 1 red and 2 green segments.

Now, re-plotting the segments obtained adding heads, I see that the difference between the segments with half number of legs and number of heads is directly the number of llamas:

L = 61 – 47 = 14

The length of the other segment could be known if I subtract from the number of heads the number of llamas' heads, and I obtain number of ostriches' heads:

O = 47 – 14 = 33

i.e. the same result.

Scoring Rubric

A **problem-specific rubric** can be found linked from the problem to help in assessing student solutions. We consider each category separately when evaluating the students' work, thereby providing more focused information regarding the strengths and weaknesses in the work.

We hope these packets are useful in helping you make the most of Pre-Algebra Problems of the Week. Please let me know if you have ideas for making them more useful.

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