



Problem of the Week Teacher Packet

Sports Weigh In

The Math Club at Dawn's school had a Math Carnival. At her booth Dawn set up three scales with baseballs, tennis balls and soccer balls, as shown in the picture below. Contestants tried to determine the weight of each kind of ball.



How much does each kind of ball weigh?

Explain how you solved it. Show how you know you are correct.

[Assume that all the soccer balls weigh the same, all the baseballs weigh the same, and all the tennis balls weigh the same.]

Extra: Find all the other combinations of these kinds of balls that would weigh 22 ounces. Explain how you found them and how you know you have found them all.

Answer Check

After students submit their solution, they can choose to “check” their work by looking at the answer that we provide. Along with the answer itself (which never explains how to actually get the answer) we provide hints and tips for those whose answer doesn't agree with ours, as well as for those whose answer does. You might use these as prompts in the classroom to help students who are stuck and also to encourage those who are correct to improve their explanation.

The baseballs weigh 5 ounces each. Now you can find the rest.

If your answer **doesn't** match ours,

- can you find how much one soccer ball and one tennis ball weigh together?
- did you try doubling the scale on the left? Can you compare that to another scale?
- did you make estimates and test them?
- did you make a list?
- did you check your arithmetic?

If you used guess and check, did you tell . . .

- what numbers you tried?
- how you tested them?
- how you knew whether they worked or not?
- how you decided what to try next?
- about any patterns that helped you?

If any of those ideas help you, you might revise your answer, and then leave a comment that tells us what you did. If you're still stuck, leave a comment that tells us where you think you need help.

If your answer **does** match ours,

- is your explanation clear and complete?
- did you try the Extra question?
- did you verify your answer with another method?
- did you show how you know your answer is right?
- did you have any "Aha!" moments? Describe them.

Revise your work if you have any ideas to add. Otherwise leave us a comment that tells us how you think you did—you might answer one or more of the questions above.

Our Solutions

Method 1: Guess and Check

[While I expect many students to use this strategy, guess and check is not as well suited to this problem as it is to many others. It is not simple to gather information from incorrect guesses and use it to make adjustments.]

I used guess and check. I compared the first and last scales and saw that a baseball must weigh 3 oz more than a tennis ball. I tried three weights that would maintain that difference and work on the first scale. Then I found out what the others would be. I made a table of how that would work. When I adjusted after each test, I kept the total of all three balls at 19 oz.

	$B + S + T = 19 \text{ oz}$	$2S + 2T = 28 \text{ oz}$	$S + 2B = 22 \text{ oz}$	
B=7 S=8 T=4	$7 + 8 + 4 = 19 \text{ oz}$	$16 + 8 = 24 \text{ oz}$	$8 + 14 = 22 \text{ oz}$	Need more weight on middle scale without changing totals of 1st and 3rd. Try heavier soccer ball, must be even number.
B=6 S=10 T=3	$6 + 10 + 3 = 19 \text{ oz}$	$20 + 6 = 26 \text{ oz}$	$10 + 12 = 22 \text{ oz}$	Getting closer. Make soccer ball heavier.
B=5 S=12 T=2	$5 + 12 + 2 = 19 \text{ oz}$	$24 + 4 = 28 \text{ oz}$	$12 + 10 = 22 \text{ oz}$	Yes!

The tennis ball weighs 2 oz, the soccer ball weighs 12 oz, and the baseball weighs 5 oz. The bottom row of my table shows that those weights are correct.

Method 2: Logical Reasoning (halving the middle scale)

Since the middle scale contains 2 soccer balls and 2 tennis balls, one soccer ball and one tennis ball together must weigh half of that, or 14 oz.

Comparing that to the 19 oz scale tells me that the baseball weighs 5 oz.

$$\text{Total} - (\text{soccer} + \text{tennis}) = 19 - 14 = 5 \text{ oz.}$$

Looking at the third scale, I can tell that the soccer ball must weigh 12 oz.

$$\text{Total} - 2 \text{ baseballs} = 22 - 10 = 12 \text{ oz.}$$

Looking back at the first scale, I can tell that the tennis ball must weigh 2 oz.

$$\text{Total} - (\text{soccer} + \text{baseball}) = 19 - 17 = 2 \text{ oz.}$$

I tested my values to make sure they worked on the scales. T stands for the weight of a tennis ball, S for a soccer ball, and B for a baseball.

$$T + S + B = 2 + 12 + 5 = 19 \text{ oz.}$$

$$2S + 2T = (2 \cdot 12) + (2 \cdot 2) = 24 + 4 = 28 \text{ oz.}$$

$$S + 2B = 12 + (2 \cdot 5) = 12 + 10 = 22 \text{ oz.}$$

Extra: I made a table to find all the other ways to make 22 oz. I started with the greatest number of soccer balls possible, then the maximum number of baseballs, and decreased systematically. I knew that any combination including baseballs must use an even number of them, because 22 is even.

Soccer @ 12 oz	Baseballs @ 5 oz	Tennis @ 2 oz	Total weight
1		5	$12 + 10 = 22$ oz
	4	1	$20 + 2 = 22$ oz
	2	6	$10 + 12 = 22$ oz
		11	$11 \cdot 2 = 22$ oz

Method 3: Logical Reasoning (doubling the first scale)

I doubled the first scale.

$$2 \text{ baseballs} + 2 \text{ soccer balls} + 2 \text{ tennis balls} = 38 \text{ oz.}$$

I compared that to the middle scale. The difference of 10 oz ($38 - 28 = 10$) must be due to the 2 baseballs, so one baseball weighs half that, or 5 oz.

I subtracted the weight of 2 baseballs from the third scale to find the weight of a soccer ball.

$$22 - 10 = 12 \text{ oz.}$$

When I subtracted the weight of a soccer ball and a baseball from the first scale, I found that a tennis ball weighs 2 oz.

$$19 - 12 - 5 = 2 \text{ oz.}$$

I tested my values to make sure they worked on the scales. I used t to stand for the weight of a tennis ball, s for a soccer ball, and b for a baseball.

$$t + s + b = 2 + 12 + 5 = 19 \text{ oz.}$$

$$2s + 2t = (2 \cdot 12) + (2 \cdot 2) = 24 + 4 = 28 \text{ oz.}$$

$$s + 2b = 12 + (2 \cdot 5) = 12 + 10 = 22 \text{ oz.}$$

It works!

Method 4: Algebra

We let S stand for the weight of a soccer ball, B for that of a baseball, and T for that of a tennis ball. We used the information from two scales to express T and B in terms of S .

$$\text{From the middle scale: } 2S + 2T = 28$$

$$\text{Divide both sides by 2: } S + T = 14$$

$$T = 14 - S$$

$$\text{From the scale on the right: } S + 2B = 22$$

$$\text{Divide both sides by 2: } S/2 + B = 11$$

$$B = 11 - S/2$$

$$\text{Substitute for } T \text{ and } B \text{ into the first scale: } S + T + B = 19$$

$$S + (14 - S) + (11 - S/2) = 19$$

$$\text{Collect like terms: } 14 + 11 - 19 = S - S + S/2$$

$$S/2 = 6$$

$$S = 12$$

$$\text{Substitute for } S \text{ on the middle scale: } 24 + 2T = 28$$

$$2T = 4$$

$$T = 2$$

Substitute for S on the right: $12 + 2B = 22$

$$2B = 10$$

$$B = 5$$

We tested our values for S, T and B on the 3 scales to check our answer:

$$5 + 12 + 2 = 19 \text{ oz}$$

$$24 + 4 = 28 \text{ oz}$$

$$10 + 12 = 22 \text{ oz}$$

Standards

If your state has adopted the [Common Core State Standards](#), you might find the following alignments helpful.

Grade 3: Operations & Algebraic Thinking

Solve two-step word problems using the four operations. Represent these problems using equations with a letter standing for the unknown quantity. Assess the reasonableness of answers using mental computation and estimation strategies including rounding.

Grade 4: Operations & Algebraic Thinking

Solve multistep word problems posed with whole numbers and having whole-number answers using the four operations, including problems in which remainders must be interpreted. Represent these problems using equations with a letter standing for the unknown quantity. Assess the reasonableness of answers using mental computation and estimation strategies including rounding.

Grade 5: Operations & Algebraic Thinking

Write simple expressions that record calculations with numbers, and interpret numerical expressions without evaluating them.

Mathematical Practices

1. Make sense of problems and persevere in solving them.
3. Construct viable arguments and critique the reasoning of others.

Teaching Suggestions

Sports Weigh In requires understanding that all soccer balls are of equal weight, as are all baseballs and all tennis balls. The goal of this problem is to develop logical reasoning, not simply arrive at an answer.

As mentioned in Method 1 above, guess and check is not the most effective way to solve the problem, but many students are confident of that approach, and will try it. You might encourage them to find an alternative approach, at least to verify their answers. Ask them to look carefully at the scales and try to notice relationships that can lead to a more direct and insightful solution. The questions in the answer check above may be useful in helping students make such discoveries.

Students who do approach the problem with a guess and check method need to be encouraged to make use of the information gained from incorrect attempts in order to make a better next guess – achieving success through skill and understanding, not pure luck. Some students might uncover ways that parity (odd/even) can help them. See the answer check, above, for more details about effective use of guess and check.

The Extra provides an opportunity for students to find a systematic method of ensuring they have found all combinations.

The Online Resources Page for this problem contains links to related problems in the Problem Library and to other web-based resources.

If you would like a calendar of the Current Problems, consider bookmarking this page:

Sample Student Solutions - Focus on *Strategy*

In the solutions below, I've provided scores the students would have received in the **Strategy** category of our scoring rubric. My comments focus on areas in which they seem to need the most improvement.

Novice	Apprentice	Practitioner	Expert
Has no ideas that will lead them toward a successful solution or shows no evidence of strategy.	Uses a strategy that uses luck instead of skill, or doesn't provide enough detail to determine whether it was luck or skill.	Uses a strategy that relies on skill, not luck, which might include: <ul style="list-style-type: none">• thorough noticing and wondering• guess and check with reasonable decisions• use logical reasoning	Does one or more of these: Uses two different strategies. Uses an unusual or sophisticated strategy.

Elijah, age 11, Novice

tennis ball weighs 4,oz baseball weighs 6,oz and soccer ball weighs 10,oz
i added all the oz of the balls together

Elijah picked one of his favorite operations and used it with the numbers at hand. Although this is a popular strategy among students it indicates Elijah just wants to be over and done!

I would encourage Elijah notice and wonder using just the Scenario. He might start making sense of the problem if the question is out of the picture (so to speak).

Michael, age 11, Novice

I added up all the weight then divided it by 3
23 is how much each ball weigh

Michael's strategy is similar to Elijah except he used two of his favorite operations instead of just one.

I would encourage Michael to join Elijah in a noticing and wondering activity.

Isabella, age 14, Novice

socer balls must weigh 12 pounds, tennis balls must weigh 2 pounds, and baseballs must weigh 5 pounds.

i wrote down the equations to calculate the weight on each scale

$$\begin{aligned}s+t+b&=19 \\ s+s+t+t&=28 \\ b+b+s&=22\end{aligned}$$

i then tried to replace the letters with numbers i could tell that the soccer ball had allot more weight then the rest and the tennis ball was allot lighter

I notice Isabella started using an algebraic approach and switched over to guess and check.

I wonder how she knew that the soccer ball had a lot more weight than the other balls. Similarly, I wonder

i tried a whole bunch of numbers and i ended up with 12 for the soccer ball, 2 for the tennis ball, and 5 for the baseball,

how she knew that the tennis ball was a lot lighter.

I would encourage her to look at her equations before switching too quickly to guess and test.

Samantha, age 9, Apprentice

The soccer ball weighs 12 ounces, the baseball weighs 5 ounces, and the tennis ball weighs 2 ounces.

Extra: 1 soccer ball and 2 baseballs; 1 soccerball and 5 tennis balls; 2 baseballs and 6 tennis balls; 4 baseballs and 1 tennis balls; and 11 tennis balls.

I used different combinations until I came up with the correct numbers. I know I am correct because I was able to use these weights for all three problems and I got the amount asked for (e.x. $12+5+2=19$, $12+12+2+2=28$, $12+5+5=22$).

Extra: Since a soccer ball weighs 12 ounces, I could only use it once in a combination. I started from that point to come up with my combinations. A baseball weighs 5 ounces, so I had to multiple it by an even number. That's why I only used 2 in one combination and 4 in the other. Lastly, I used the tennis ball combination by dividing 22 by 2 and making my combinations with the tennis ball.

Samantha verified her solution by showing the sums of the weights on each scale using her solution. We can't tell whether she applied her guess and check approach systematically or randomly or how she made decisions as she tested. I'd ask for an example of a trial that didn't work. She did make insightful observations in the Extra.

Lise, age 8, Practitioner

My answers are that the soccerball weighs 12 oz.,the tennisball is 2 oz.,and the baseball is 5 oz.

First I added $13 + 13 = 26$ to try to find out what the soccerball weighed.I was looking at the middle weight.Then I saw that the tennisball weighed 1 oz.if I did it that way. I thought that wouldn't be enough oz.So next I tried 12 oz. $12 + 12 = 24$.That would leave me 4 oz.That would be enough for 2 tennisballs because each would be 2 oz.To check it I tried it in the first weight and the 3rd weight.First I tried it in the first weight. I saw a soccerball,a baseball,and a tennisball.I tried adding my answers for the soccerball and tennisball. $2 + 12 = 14$ oz.I was trying to get up to 19 oz.so then $19 - 14 = 5$ oz.Then the baseball would equal 5 oz.Then to check my answers again I went to the third weight and saw 1 soccerball and 2 baseballs. $12 \text{ oz (soccerball)}+5 \text{ oz.(baseball)}+ 5 \text{ oz. (another baseball)}= 22 \text{ oz.}$ wich is the total weight that I'm trying to get to.That is how I got my answers and checked my answers.

Lise described a systematic guess and check, giving reasons for choosing a starting number and adjusting it. She told how she tested her guess and how she knew it was right. One space between sentences would improve readability. I think she's ready to try the Extra.

Anna, age 9, Practitioner

The tennis balls weigh 2oz. The Soccer ball weighs 12oz. Baseballs weigh 5oz.

To find my answer I knew that 2 soccer balls and 2 tennis balls were 28oz. So, to the weight of 1 soccer ball and 1 tennis ball, you would have to divide 28 by 2 getting you 14oz.

I also knew that 1 baseball, 1 soccer ball and 1 tennis ball weighed 19oz. Knowing that 14oz were taken up by the soccer ball and tennis ball, what ever was left must have been the weight of the baseball. So, I subtracted 14 from 19 and got 5oz as the weight of the base ball. So if the balls weighed 14oz. together plus 5 that would be 19oz.

Also knowing that 2 baseballs and 1 soccer ball was equal to the weight of 22oz, and knowing that each baseball was 5oz, 2 of them would equal 10oz,

Anna's articulate explanation of logical reasoning shows clear thinking and an ability to see the relationships among the balls and weights. It would be interesting to see how she would apply her reasoning ability to solve the Extra.

and the rest would be the weight of the soccer ball. So then I subtracted 10oz from 22oz, and got 12oz as the weight of the soccer ball.

Then, if the soccer ball was 12oz and the baseball was 5oz and along with the tennis ball, the whole weight was 19oz, I knew that subtracting 17oz. from 19oz. Leaving 2 ounces for the weight of the soccer ball. So 12oz. plus 5oz. plus 2oz. Is 19oz

Archit, age 11, Expert

The soccer ball is 12 oz, the baseball 5 oz, and the tennis is 2 oz. To find it, suppose s is the weight of the soccer ball, t is the weight of the tennis ball, and b is the weight of the baseball. The following are given:

- I. $b+s+t=19$
- II. $2s+2t=28$
- III. $2b+s=22$

II can be reduced to $s+t = 14$. That can be fitted into $b+s+t=19$, making it $b+14=19$. That solves to $b=5$. From that, fit the value of b into III, so $2(5) + s = 22$ or $s = 12$. Last, fit $s=12$ into $s+t=14$. That results in $t=2$.

EXTRA: First, there can only be 1 or 0 soccer balls, because 2 soccer balls weigh 24 oz which exceeds 22 oz. With 1 soccer ball, the combination of tennis and baseballs must be 10. Therefore, there can be 5 tennis balls or 2 baseballs. With 0 soccer balls, the sum of the baseballs and tennis balls must be 22. There can not be more than 4 baseballs, since 5 baseballs weigh 25, which is more than 22. Also, the number of baseballs must be even, since otherwise we can not make up the difference with tennis balls. So, there are only three possibilities, 4 baseballs, 2 baseballs or no baseballs. With 4 baseballs there is one tennis ball needed. With 2 baseballs there are 6 tennis balls and with 0 baseballs there are 11 tennis balls. So, there are 5 possible combinations and no others:

- 1 soccer ball, 5 tennis balls
- 1 soccer ball, 2 baseballs
- 4 baseballs, 1 tennis ball
- 2 baseballs, 6 tennis balls
- 11 tennis balls.

Anne, age 10, Expert

Each soccer ball is 12 oz. Each baseball is 5 oz. Each tennis ball is 2 oz. Extra: Other combinations can be 11 tennis balls, 4 baseballs and 1 tennis ball, 1 soccer ball and 2 baseballs, 1 soccer ball and 5 tennis balls, and 2 baseballs and 6 tennis balls.

I first looked at the scale in the middle. There are 2 soccer balls and 2 tennis balls. I already know 1 soccer ball and 1 tennis ball weigh 14 oz. Because $28/2=14$

Then I looked at the first scale. There is 1 baseball, 1 soccer ball, and 1 tennis ball. 1 soccer ball and 1 tennis ball equal 14 oz. Since $19 \text{ oz.} - 14 \text{ oz.} = 5 \text{ oz.}$ Therefore 1 baseball weigh 5 oz.

Archit's algebraic approach earns Expert in Strategy. He provides clear rationale to support his finding all 22 oz Extra combinations.

Anne's extra approach is very analytical and systematic, earning Expert status.

At last, I looked at the last scale. There are 2 baseballs and 1 soccer ball. Each baseball weighs 5 oz. $5+5=10$ and $22-10=12$. So each soccer ball weighs 12 oz. Now I know each tennis ball weighs 2 oz. because $14-12=2$.

Extra: In order to make the balls equal to 22 oz., there are 5 possibilities:

- (1) 1 soccer and 2 base balls.
- (2) 1 soccer and 5 tennis balls.
- (3) 4 base and 1 tennis balls.
- (4) 2 base and 6 tennis balls
- (5) 11 tennis balls.

Explanation: There are a maximum of 7 possibilities. They are:

- (1) has all three kinds of balls
- (2) has only soccer and base balls
- (3) has only soccer and tennis balls
- (4) has only base and tennis balls
- (5) has only soccer balls
- (6) has only base balls
- (7) has only tennis balls.

In order to make the balls equal to 22 oz., there is no answer to (1), there is one answer to (2), 1 answer to (3), two answers to (4), no answer to (5) and (6), and one answer to (7).

Scoring Rubric

A **problem-specific rubric** can be found linked from the problem to help in assessing student solutions. We consider each category separately when evaluating the students' work, thereby providing more focused information regarding the strengths and weaknesses in the work.

We hope these packets are useful in helping you make the most of the Math Fundamentals Problems of the Week. Please let me know if you have ideas for making them more useful.

<https://www.nctm.org/contact-us/>