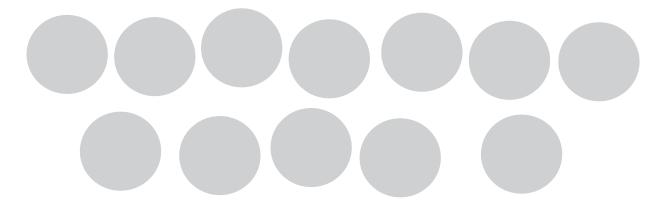
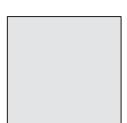
Student Sheet 1

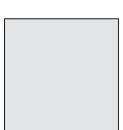
1. If 3 people share 12 cookies equally, how many cookies does each person get?

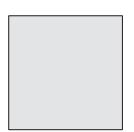


2. Four people want to share 5 cakes equally. Show how much each person gets.





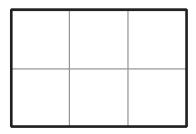




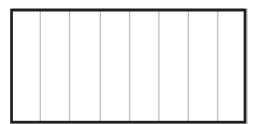


Student Sheet 2

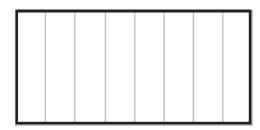
1. The candy bar below can be shared equally by 6 people. Shade the amount that 1 person gets. What fraction does each person get?



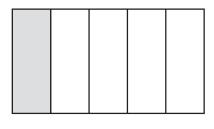
2. Shade 1/8 (one-eighth) of the candy bar.

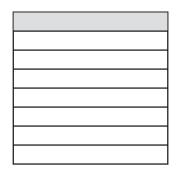


3. Shade 1/4 (one-fourth) of the candy bar.



4. Use fractions to tell how much of each candy bar is shaded.





В

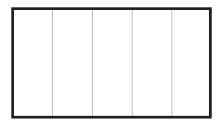


C

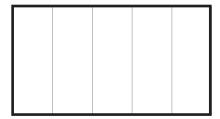
Why is C *not* one-half?

Student Sheet 3

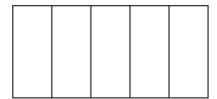
1. The candy bar below has been divided up so that it can be shared equally by 5 people. Shade the amount that 2 people get. What fraction of the candy bar is shaded?



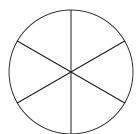
2. The candy bar below has been divided up so that it can be shared equally by 5 people. Shade the amount that 4 people get. What fraction of the candy bar is shaded?



3. Shade 3/5 of the candy bar below.

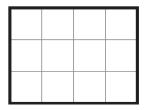


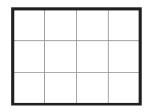
4. Shade 5/6 of the pizza below.

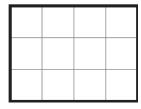


Student Sheet 4

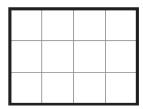
1. Shade each black rectangle below to show one-half (1/2) in a different way. For each of your answers, tell how you know that one-half of the black rectangle is shaded.

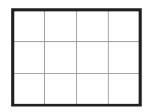


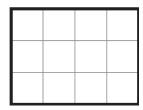




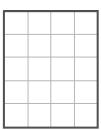
2. Shade each black rectangle below to show one-third (1/3) in a different way. Tell how you know that one-third of the black rectangle is shaded.

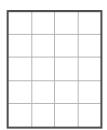


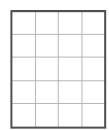




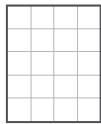
3. Shade each black rectangle below to show one-fourth (1/4) in a different way. Tell how you know that one-fourth of the black rectangle is shaded.

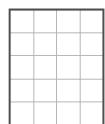


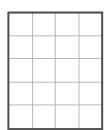




4. Shade each black rectangle below to show one-tenth (1/10) of the rectangle in a different way. Tell how you know that one-tenth of the black rectangle is shaded.



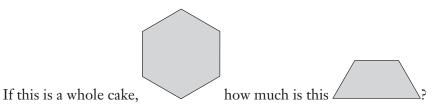


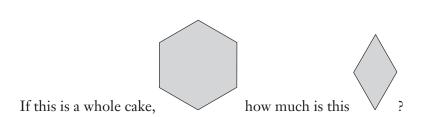


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Student Sheet 5

Use pattern blocks to find answers to each question. Explain why you think your answers are correct.



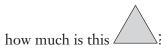


Student Sheet 6

Use pattern blocks to find answers to each question.



If this is a whole cake,





If this is a whole cake,



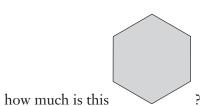
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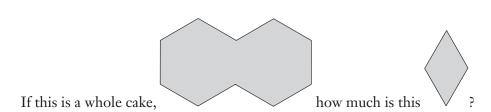
Student Sheet 7

Use pattern blocks to find answers to each question.



If this is a whole cake,



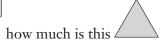


Student Sheet 8

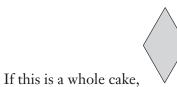
Use pattern blocks to find answers to each question.



If this is a whole cake,

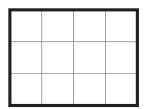


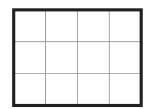
If this is a whole cake, 4

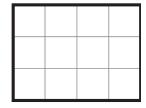


Student Sheet 9

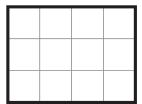
1. Shade each black rectangle below to show two-thirds (2/3) of the rectangle in a different way. Tell how you know that two-thirds of the black rectangle is shaded.

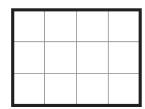


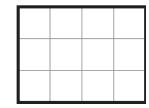




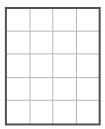
2. Shade each black rectangle below to show three-fourths (3/4) of the rectangle in a different way. Tell how you know that three-fourths of the black rectangle is shaded.

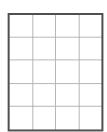


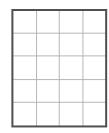




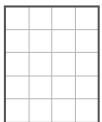
3. Shade each black rectangle below to show four-fifths (4/5) of the rectangle in a different way. Tell how you know that four-fifths of the black rectangle is shaded.

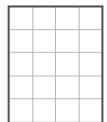


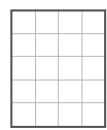




4. Shade each black rectangle below to show three-tenths (3/10) of the rectangle in a different way. Tell how you know that three-tenths of the black rectangle is shaded.



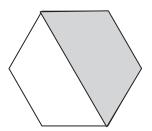


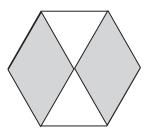


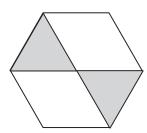
Student Sheet 10

Tell what fraction of each shape is shaded. Predict, then check with pattern blocks.

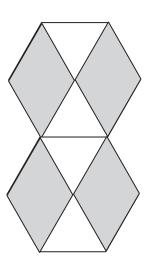
1.

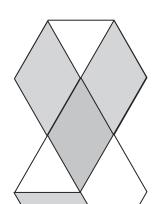


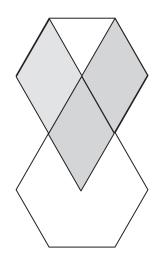


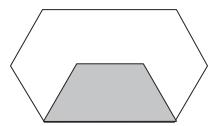


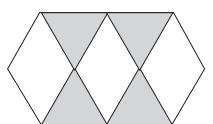
2.







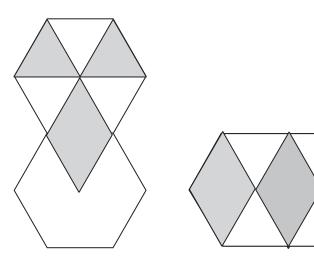


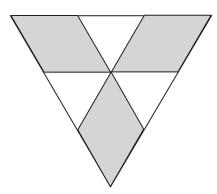


Student Sheet 11

Tell what fraction of each shape is shaded. Predict, then check with pattern blocks.

1.



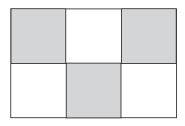


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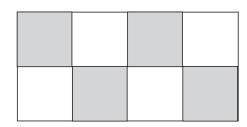
Student Sheet 12

Tell what fraction of each rectangle is shaded. Predict, then check with pattern blocks.

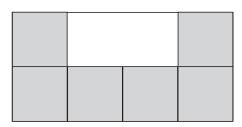
1.

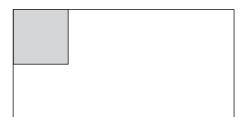


2.



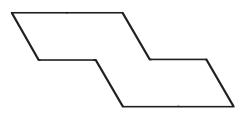
3.





Student Sheet 13

What fraction of the large shape does each shaded region cover? Predict, then check with pattern blocks.



1.

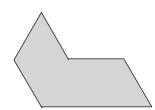


2.

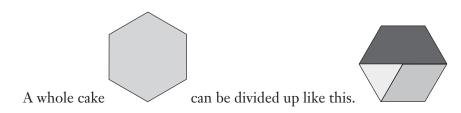


3.





Student Sheet 14



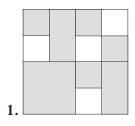
Is the triangle one-third of the whole? Explain.

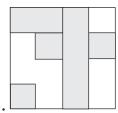
Is the trapezoid one-third of the whole? Explain.

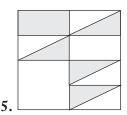
Is the rhombus one-third of the whole? Explain.

Student Sheet 15

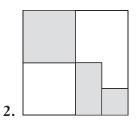
For each shape, tell what fraction of the shape is shaded. How would you convince the other students in your class that your answer is correct?

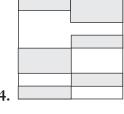


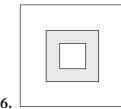












Name	Date
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Student Sheet 16

Use Cuisenaire rods. Can you find answers that are just one rod?

1. Find 1/5 if the orange rod is 1. Prove it is 1/5. [Use the Cuisenaire rods.]

Find 2/5 if the orange rod is 1. Prove it is 2/5.

Find 3/5 if the orange rod is 1. Prove it is 3/5.

Explain how you found your answers [in writing].

2. Find 1/4 if the brown rod is 1. Prove it is 1/4.

Find 2/4 if the brown rod is 1. Prove it is 2/4.

Find 3/4 if the brown rod is 1. Prove it is 3/4.

3. Find 1/6 if the dark green rod is 1. Prove it is 1/6.

Find 2/6 if the dark green rod is 1. Prove it is 2/6.

Find 5/6 if the dark green rod is 1. Prove it is 5/6.

4. Find 1/7 if the black rod is 1. Prove it is 1/7.

Find 3/7 if the black rod is 1. Prove it is 3/7.

Find 4/7 if the black rod is 1. Prove it is 4/7.

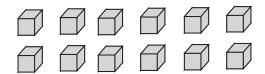
5. Find 1/9 if the blue rod is 1. Prove it is 1/9.

Find 4/9 if the blue rod is 1. Prove it is 4/9.

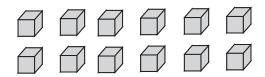
Find 7/9 if the blue rod is 1. Prove it is 7/9.

Student Sheet 17

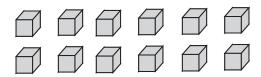
1. How many cubes are in 1/2 of a bag of 12 cubes? Explain your answer.



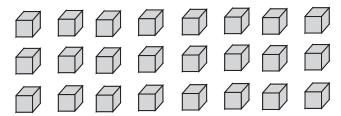
2. How many cubes are in 1/3 of a bag of 12 cubes? Explain your answer.



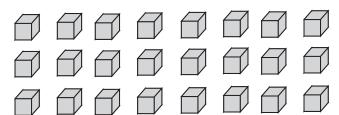
3. How many cubes are in 1/4 of a bag of 12 cubes? Explain your answer.



4. How many cubes are in 1/2 of a bag of 24 cubes? Explain your answer.

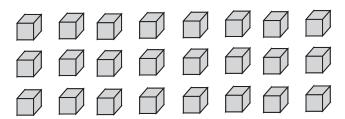


5. How many cubes are in 1/3 of a bag of 24 cubes? Explain your answer.

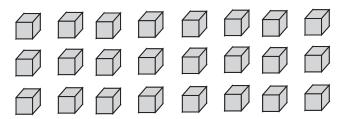


Student Sheet 18

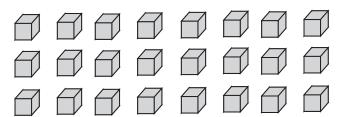
1. How many cubes are in 1/4 of a bag of 24 cubes? Explain your answer.



2. How many cubes are in 1/8 of a bag of 24 cubes?

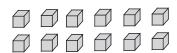


3. How many cubes are in 1/6 of a bag of 24 cubes? Explain your answer.

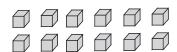


Student Sheet 19

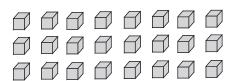
1. How many cubes are in 3/4 of a bag of 12 cubes? Explain your answer.



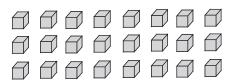
2. How many cubes are in 5/6 of a bag of 12 cubes? Explain your answer.



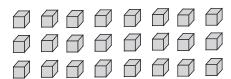
3. How many cubes are in 5/6 of a bag of 24 cubes? Explain your answer.



4. How many cubes are in 3/4 of a bag of 24 cubes? Explain your answer.



5. How many cubes are in 5/12 of a bag of 24 cubes? Explain your answer.

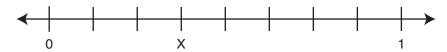


Name		Date	
CBA Fractions			
Student Sheet 20			
1. Shade 1/3 of the squares. How ma	any squares o	did you sh	nade? Explain your answer.
2. Shade 2/3 of the squares. How ma	any squares (did you sh	nade? Explain your answer.
3. Shade 5/6 of the squares. How ma	nny squares o	did you sh	nade? Explain your answer.
4. Shade 3/4 of the squares. How ma	any squares	did you sh	nade? Explain your answer.
		1 —	

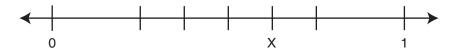
May be photocopied for classroom or workshop use. © 2012 by Michael Battista from Cognition–Based Assessment and Teaching of Fractions: Building on Students' Reasoning. Portsmouth, NH: Heinemann.

Student Sheet 21

1. Name the fraction marked by X on the number line. Explain your answer.



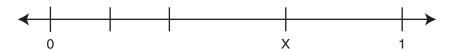
2. Name the fraction marked by X on the number line. Explain your answer.



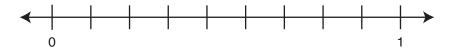
3. Place an X at 3/4 on the number line. Explain your answer.



4. Name the fraction marked by X on the number line. Explain your answer.



5. Place an X at 2/3 on the number line. Explain your answer.



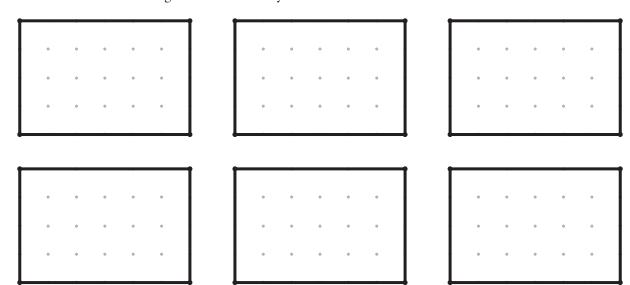
6. Name the fraction marked by X on the number line. Explain your answer.



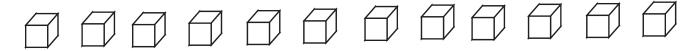
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Student Sheet 22

1. Shade 1/2 of the rectangle in different ways to show that 1/2 = 2/4 = 3/6 = 4/8 = 6/12 = 12/24.



2. Show 2/3 of this set of cubes.



Show 4/6 of this set of cubes.



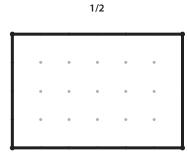
Show 8/12 of this set of cubes.

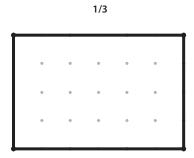


Explain how what you did above shows that 2/3 = 4/6 = 8/12.

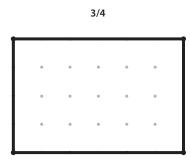
Student Sheet 23

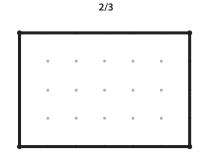
1. Use the picture to decide which number, 1/2 or 1/3, is larger or if they are equal. Explain your answer.



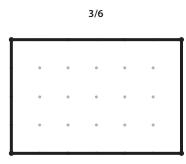


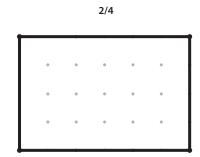
2. Use the picture to decide which number, 3/4 or 2/3, is larger or if they are equal. Explain your answer.





3. Use the picture to decide which number, 3/6 or 2/4, is larger or if they are equal. Explain your answer.

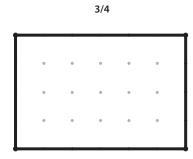




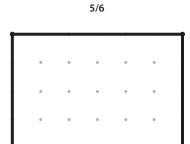
Student Sheet 23 (Continued)

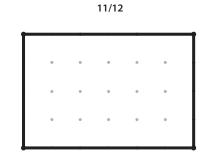
4. Use the picture to decide which number, 5/6 or 3/4, is larger or if they are equal. Explain your answer.





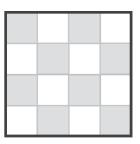
5. Use the picture to decide which number, 5/6 or 11/12, is larger or if they are equal. Explain your answer.



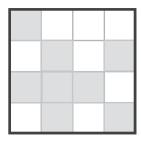


Student Sheet 24

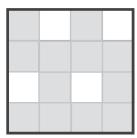
1. How many small squares are there in the shape below? How many small squares are shaded? What fraction of the shape below is shaded? What other fractions name the amount of the shape below that is shaded? How can you prove your answers correct (a) with pictures, (b) with numbers?



2. How many small squares are there in the shape below? How many small squares are shaded? What fraction of the shape below is shaded? What other fractions name the amount of the shape below that is shaded? How can you prove your answers correct (a) with pictures, (b) with numbers?



3. How many small squares are there in the shape below? How many small squares are shaded? What fraction of the shape below is shaded? What other fractions name the amount of the shape below that is shaded? How can you prove your answers correct (a) with pictures, (b) with numbers?



Student Sheet 25

1. If this rod is 1/5, find the rod that is 1. Show how you can prove that your answer is correct.

Red

2. If this rod is 1/3, find the rod that is 1. Show how you can prove that your answer is correct.

Light Green

3. If this rod is 2/3, find the rod that is 1. Show how you can prove that your answer is correct.

Dark Green

4. If this rod is 2/5, find the rod that is 1. Show how you can prove that your answer is correct.

Purple

CHALLENGE PROBLEM

5. If this rod is 1/3, find the rod that is 5/6. Show how you can prove that your answer is correct.

Red

Name	Date
CBA Fractions	
Student Sheet 26	
Solve each problem with Cuisenaire rods. Explain h	ow you find your answers.
1. If the red rod is 1/4, what rod is 1? Prove it.	
2. If the white rod is 1/5, what rod is 1? Prove it.	
3. If the light green rod is 1/3, what rod is 1? Pro	we it
3. If the light green rod is 17.3, what rod is 1. 110	ve ic
4.76.1.1.1.1.275.1.1.1.43.0.	
4. If the dark green rod is 3/5, what rod is 1? Pro	ve it.
5. If the purple rod is 4/5, what rod is 1? Prove it	
6. If the yellow rod is 5/8, what rod is 1? Prove it	

M	D-4-
Name	Date
Name	Date

Student Sheet 27

For each problem, a fractional part of the whole is shown. Find the whole. Explain.

1/4	1/4	1/4
2/3	2/3	2/3
1/3	1/5	1/7
1/3	1/5	1/7
		1/7

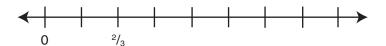
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Student Sheet 28

- 1. If 3 cubes is 1/5 of a bag of cubes, how many cubes are in the whole bag?
- 2. If 2 cubes is 1/4 of a bag of cubes, how many cubes are in the whole bag?
- 3. If 6 cubes is 2/3 of a bag of cubes, how many cubes are in the whole bag?
- **4.** If 6 cubes is 3/7 of a bag of cubes, how many cubes are in the whole bag?
- **5.** Jon ate 12 cookies. That was 3/4 of the whole bag of cookies. How many cookies were in the whole bag?

CHALLENGE PROBLEMS

- **6.** Mario ate 40 jelly beans. That was 5/8 of the whole bag of jelly beans. Naomi ate 1/4 of the whole bag of jelly beans. How many jelly beans did Naomi eat? What fraction of the whole bag of jelly beans was left?
- 7. Place an X at 1, a Y at 2, and a Z at 2 1/3 on the number line below. Explain your answers.



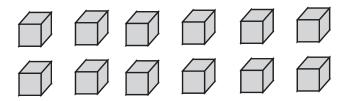
Nama a	Data
Name	Date
tarric	Date

Student Sheet 29

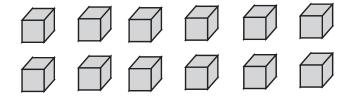
The rectangle is 1. Show 2.	The rectangle is 1. Show 1 1/2.
The rectangle is 1. Show 3/2.	The rectangle is 1. Show 5/4.
The rectangle is 1. Show 7/4.	The rectangle is 1. Show 13/6.

Student Sheet 30

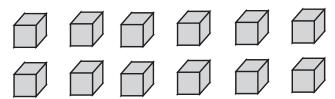
1. How many cubes are in 3/2 of a bag of 12 cubes? Explain your answer.



2. How many cubes are in 5/2 of a bag of 12 cubes? Explain your answer.



3. How many cubes are in 7/4 of a bag of 12 cubes? Explain your answer.



Name	Date

Student Sheet 31

For each problem, first try to figure out the answer mentally. Then check your answer by drawing pictures or using cubes. [You can snap cubes together to prove your answers.]

1. What is 1/3 of 3 cubes?

What is 1/3 of 6 cubes?

What is 1/3 of 12 cubes?

What is 1/3 of 24 cubes?

Write a description for a procedure you can use to find 1/3 of a number. Use your procedure to find 1/3 of 36.

2. What is 2/3 of 3 cubes?

What is 2/3 of 6 cubes?

What is 2/3 of 12 cubes?

What is 2/3 of 24 cubes?

Write a description for a procedure you can use to find 2/3 of a number. Use your procedure to find 2/3 of 36.

3. What is 1/5 of 5 cubes?

What is 1/5 of 10 cubes?

What is 1/5 of 20 cubes?

Write a description for a procedure you can use to find 1/5 of a number. Use your procedure to find 1/5 of 30.

4. What is 3/5 of 5 cubes?

What is 3/5 of 15 cubes?

What is 3/5 of 20 cubes?

Write a description for a procedure you can use to find 3/5 of a number. Use your procedure to find 3/5 of 30.

5. Use the procedure you developed in Problems 1–4 to find 3/8 of 40. Write a description for a procedure you can use to find 3/8 of a number.

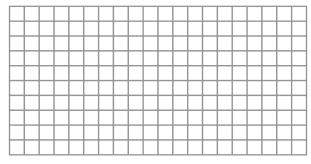
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Student Sheet 32

Explain and show what 3/5 of 15 is using each of the materials below. Your explanations should convince other people that your answers are correct. What is the same about how you solve these problems?

a. Show what 3/5 of 15 is using linking cubes. How can joining groups of cubes help you show your answer is correct? Explain how you used the cubes below.

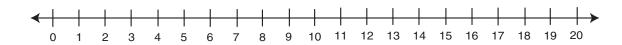
b. Show what 3/5 of 15 is using part of the graph paper below.



c. Show what 3/5 of 15 is by making your own drawing.

d. Show what 3/5 of 15 is using Cuisenaire rods. Explain how you used the rods below.

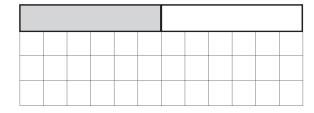
e. Show what 3/5 of 15 is using the number line below. [Hint: Think about hops along the line.]



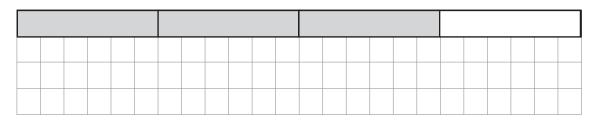
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Student Sheet 33

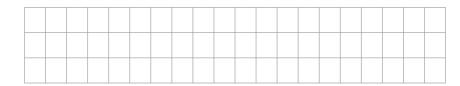
1. Draw 3 fractions equivalent to 1/2. Label each fraction you make.



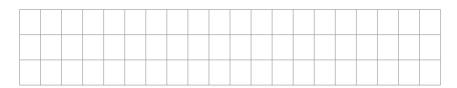
2. Draw 3 fractions equivalent to 3/4. Label each fraction you make.



3. Use pictures to find a fraction that is equivalent to 3/5 and has a denominator of 20.

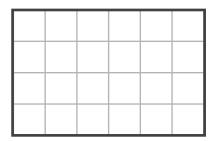


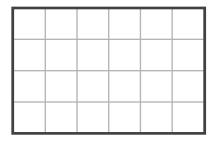
4. Use pictures to find a fraction that is equivalent to 7/9 and has a denominator of 18.



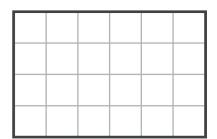
Student Sheet 34

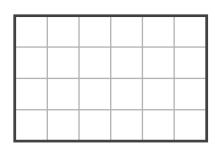
1. Use the rectangles to show which fraction is larger by converting 5/6 and 3/4 to equivalent fractions with the same denominator.



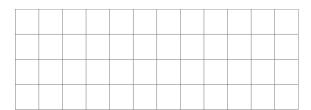


2. Use the rectangles to show which fraction is larger by converting 11/12 and 7/8 to equivalent fractions with the same denominator.



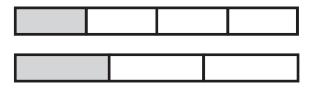


3. Draw pictures on the graph paper to show which fraction is larger by converting 2/3 and 3/4 to equivalent fractions with the same denominator.



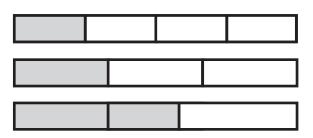
Student Sheet 35

1. Asked to find 1/4 + 1/3, Harry drew the picture at the right and said, "1 of 4 parts, plus 1 of 3 parts equals 2 of 7 parts. So the answer is 2/7."



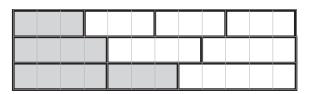
Do you think that 1/4 + 1/3 equals 2/7? Explain why or why not.

2. Asked to find 1/4 + 1/3, Harriet drew the picture at the right and said, "This is 1/4 [top]. This is 1/3 [middle]. Put 1/4 and 1/3 together [bottom]. So the answer is 2/3."



Do you think that 1/4 + 1/3 equals 2/3? Explain why or why not.

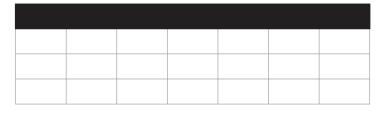
3. Asked to find 1/4 + 1/3, Kelly drew the picture at the right and said, "This is 1/4 [top] or 3/12. This is 1/3 [middle] or 4/12. Put them together [bottom] and you get 7/12."



Do you think that 1/4 + 1/3 equals 7/12? Explain why or why not.

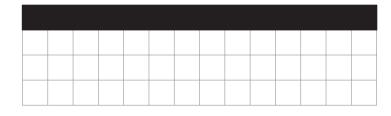
Student Sheet 36

1. Use the drawing below to find $\frac{2}{7} + \frac{3}{7}$.



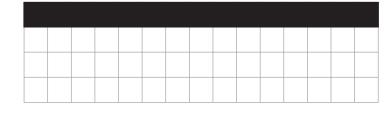
1
$$2/7$$
 $3/7$
 $2/7 + 3/7 =$

2. Use the drawing below to find $\frac{5}{14} + \frac{3}{7}$.



$$\frac{5}{14}$$
 $\frac{3}{7}$
 $\frac{5}{14} + \frac{3}{7} =$

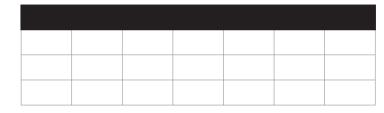
3. Use the drawing below to find $\frac{1}{3} + \frac{2}{5}$.



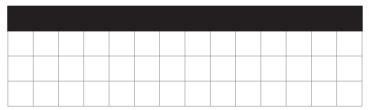
$$\frac{1}{3}$$
 $\frac{2}{5}$
 $\frac{1}{3} + \frac{2}{5} =$

Student Sheet 37

1. Use the drawing below to find $\frac{5}{7} - \frac{3}{7}$.

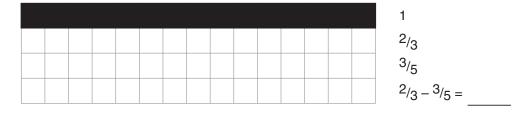


2. Use the drawing below to find $\frac{9}{14} - \frac{3}{7}$.



$$\begin{array}{c}
1 \\
9/_{14} \\
3/_{7} \\
9/_{14} - 3/_{7} = \underline{}
\end{array}$$

3. Use the drawing below to find $\frac{2}{3} - \frac{3}{5}$.



Student Sheet 38

How can getting a common denominator help you solve these problems?

PATTERN BLOCKS



1. Let the hexagon be one.



Then the rhombus

is 1/3 (why?), and the trapezoid \angle



Use pattern blocks to find 1/2 + 1/3. Explain your method.

Use pattern blocks to find 1/2 - 1/3. Explain your method.

CUISENAIRE RODS

2. Use Cuisenaire rods to find 1/2 + 1/4 and 1/2 - 1/4. [Hint. Use the dark brown rod as 1.] Explain your methods.

Nama	
Name Date	

Student Sheet 39

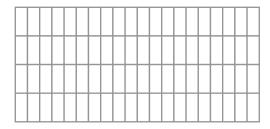
For each problem, find a common denominator to help you add or subtract the fractions. What is the lowest common denominator you can use?

Use the rectangle below to find 1/4 + 1/4.	Use the rectangle below to find 1/6 + 3/6.					
Use the rectangle below to find 1/3 + 1/4.	Use the rectangle below to find 1/3 – 1/4.					
Use the rectangle below to find 2/3 + 1/6.	Use the rectangle below to find 2/3 – 1/6.					
Use the rectangle below to find 5/12 + 1/6.	Use the rectangle below to find 5/12 – 1/6.					

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Student Sheet 40

Problem. $\frac{2}{3} + \frac{2}{5}$



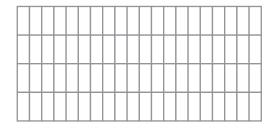
Draw the whole, or 1.

Draw the first fraction.

Draw the second fraction.

Draw the sum or difference.

Problem. $\frac{2}{3} - \frac{2}{5}$



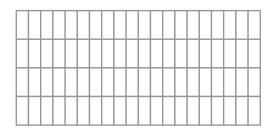
Draw the whole, or 1.

Draw the first fraction.

Draw the second fraction.

Draw the sum or difference.

Problem. $\frac{1}{4} + \frac{2}{5}$



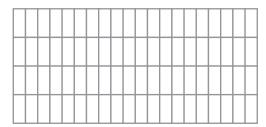
Draw the whole, or 1.

Draw the first fraction.

Draw the second fraction.

Draw the sum or difference.

Problem. $\frac{2}{5} - \frac{1}{4}$



Draw the whole, or 1.

Draw the first fraction.

Draw the second fraction.

Draw the sum or difference.

Student Sheet 41

Use *drawings* to solve the problems.

1.
$$\frac{2}{3} \times 12 = \frac{2}{3}$$
 of $12 = \underline{}$

2.
$$\frac{3}{7} \times 21 = 3/7 \text{ of } 21 = \underline{}$$

3.
$$\frac{1}{2} \times \frac{2}{3} = 1/2 \text{ of } 2/3 = \underline{\hspace{1cm}}$$

4.
$$\frac{2}{3} \times \frac{6}{7} = 2/3 \text{ of } 6/7 = \underline{}$$

5.
$$3 \times 4\frac{1}{3} = 3$$
 groups of $4\frac{1}{3} =$

6.
$$5 \times 3\frac{1}{2} = 5$$
 groups of $3\frac{1}{2} =$ _____

Student Sheet 42

Use *drawings* to solve the problems.

1.
$$3 \div \frac{1}{5}$$
 = the number of 1/5s in 3 = _____

2.
$$5\frac{1}{2} \div \frac{1}{4}$$
 = the number of 1/4s in $5\frac{1}{2}$ = _____

3.
$$6 \div \frac{2}{3}$$
 = the number of 2/3s in 6 = _____

4.
$$4\frac{1}{2} \div 3 = \text{how much in one part if } 4\frac{1}{2} \text{ is divided equally into } 3 \text{ parts} = \underline{}$$

5.
$$6\frac{2}{3} \div 4$$
 = how much in one part if $6\frac{2}{3}$ is divided equally into 4 parts = _______ [*Hint. Think of* $6\frac{2}{3}$ *as* $4\frac{8}{3}$.]

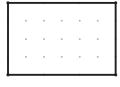
Student Sheet 43

1. Use the rectangles to find $5 \times 1/2$. [One rectangle should represent 1 whole or 1.]









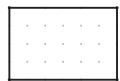


Use the rectangles to find $2 \times 3/4$. [One rectangle should represent 1 whole or 1.]





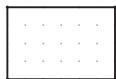
Use the rectangles to find 1/2 of 3. [One rectangle should represent 1 whole or 1.]







Use the rectangles to find $3 \times 1/2$. [One rectangle should represent 1 whole or 1.]





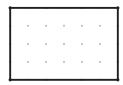


2. Use Cuisenaire rods to find $1/4 \times 2$. Explain how you found your answer.

Student Sheet 44

1. Use the rectangles to find $3 \div 3/4$. [One rectangle should represent 1 whole or 1.]



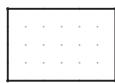




Use the rectangle to find $2/3 \div 1/6$. [One rectangle should represent 1 whole or 1.]



Use the rectangles to find 1 $1/2 \div 1/4$. [One rectangle should represent 1 whole or 1.]





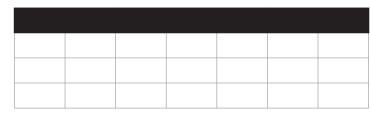
2. Use Cuisenaire rods to find 1 $1/2 \div 1/4$. Explain how you found your answer.

3. Use Cuisenaire rods to find $4/3 \div 2$. Explain how you found your answer.

Student Sheet 45

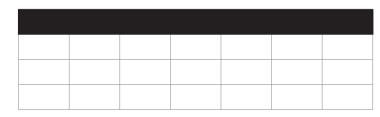
Use *drawings* to solve the problems. In each problem, the two numbers added or subtracted have the same denominator. Describe a procedure for adding or subtracting two numbers with the same denominator.

1. Use the drawing below to find $\frac{2}{7} + \frac{3}{7}$.



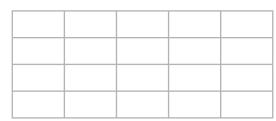
1 2/7 3/7 2/7 + 3/7 =

2. Use the drawing below to find $\frac{5}{7} - \frac{3}{7}$.

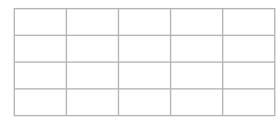


1 5/7 3/7 5/7 - 3/7 =

3. Use graph paper to find $\frac{1}{5} + \frac{2}{5}$.



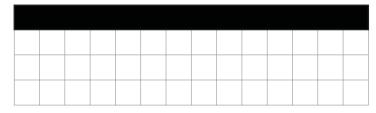
4. Use graph paper to find $\frac{4}{5} - \frac{1}{5}$.



Student Sheet 46

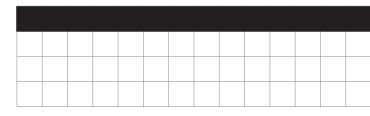
Use *drawings* to solve the problems. In each problem, the denominator of one of the numbers is a multiple of the denominator of the other number. Describe a procedure for adding or subtracting two numbers in which the denominator of one of the numbers is a multiple of the denominator of the other number.

1. Use the drawing below to find $\frac{5}{14} + \frac{3}{7}$.



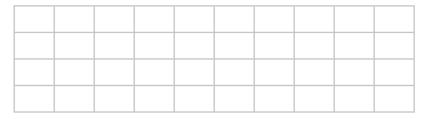
$$\frac{5}{14}$$
 $\frac{3}{7}$
 $\frac{5}{14} + \frac{3}{7} =$

2. Use the drawing below to find $\frac{9}{14} - \frac{3}{7}$.

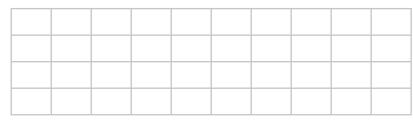


$$9/14$$
 $3/7$
 $9/14 - 3/7 =$

3. Use graph paper to find $\frac{3}{10} + \frac{2}{5}$.



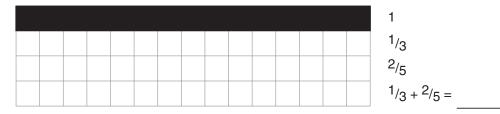
4. Use graph paper to find $\frac{4}{5} - \frac{3}{10}$.



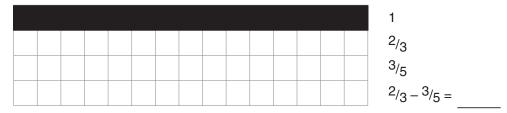
Student Sheet 47

Use *drawings* to solve the problems. Describe a procedure for adding or subtracting two numbers that will work for any denominators.

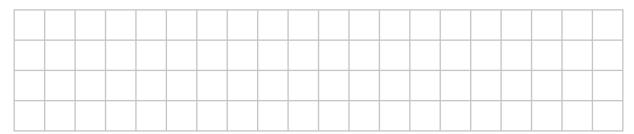
1. [Problems with unlike denominators] Use the drawing below to find $\frac{1}{3} + \frac{2}{5}$.



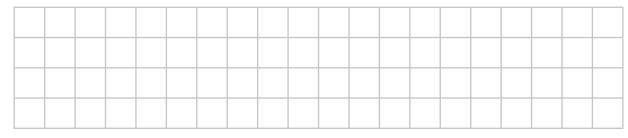
2. [Problems with unlike denominators] Use the drawing below to find $\frac{2}{3} - \frac{3}{5}$.



3. Use graph paper to find $\frac{1}{4} + \frac{2}{3}$. Decide how many squares you should put in 1.



4. Use graph paper to find $\frac{4}{5} - \frac{3}{4}$. Decide how many squares you should put in 1.



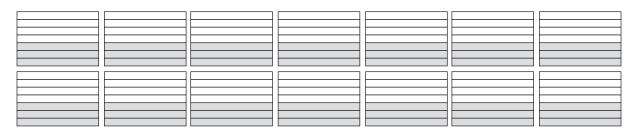
Student Sheet 48

Use *drawings* to solve the problems. Find a pattern for how to manipulate the numerators and denominators of the factors to find the numerator and denominator of the product. Write all numbers as fractions, not whole numbers or mixed numerals. Do more problems if you don't see a pattern. Describe the pattern you find.

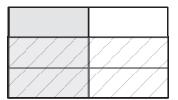
1.
$$\frac{2}{3} \times \frac{6}{1} =$$



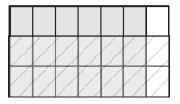
2.
$$\frac{3}{7} \times \frac{14}{1} =$$



3.
$$\frac{1}{2} \times \frac{2}{3} =$$

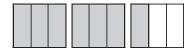


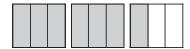
4.
$$\frac{2}{3} \times \frac{6}{7} =$$



Student Sheet 48 (Continued)

5.
$$\frac{3}{1} \times \frac{7}{3} =$$



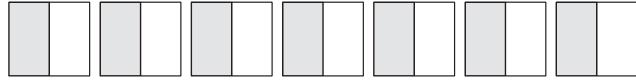


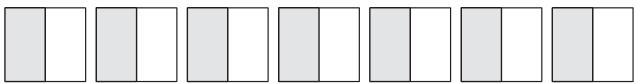


6.
$$\frac{4}{1} \times \frac{7}{2} =$$





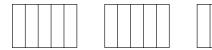




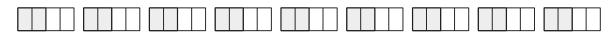
Student Sheet 49

Use *drawings* to solve the problems. Find a pattern for how to manipulate the numerators and denominators of the dividend and divisor to find the numerator and denominator of the quotient. Write all numbers as fractions, not whole numbers or mixed numerals. Do more problems if you don't see a pattern. Describe the pattern you find.

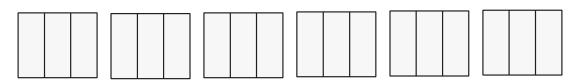
1.
$$3 \div \frac{1}{5} =$$



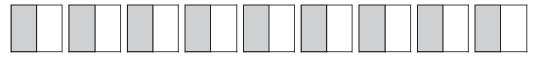
2.
$$\frac{9}{2} \div \frac{1}{4} =$$



3.
$$\frac{6}{1} \div \frac{2}{3} =$$



4. $\frac{9}{2} \div 3 =$ [Think of each of 9 halves being equally divided among 3 people. How much does each person get?]

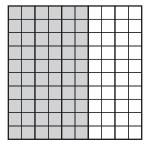


5. $\frac{8}{3} \div \frac{4}{1} =$ [Think of each of 8 thirds being equally divided among 4 people. How much does each person get?]



Student Sheet 50

1. Explain how to find 3/5 of 100 (a) using pictures, and (b) using numbers.



2. Find 3/4 of 1000. Describe how you found your answer.

3. Find 2/3 of 369. Describe how you found your answer.

4. Use numbers and pictures to find 6/5 of 25. Describe how you found your answer.

5. Use numbers and place-value blocks to find 8/5 of 50. Describe how you found your answer.

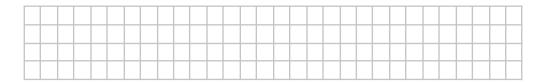
6. Find 8/5 of 100. Describe how you found your answer.

Student Sheet 51

1. Use graph paper to find 2 fractions equivalent to 3/4. Explain what you did with your graph paper pictures. Describe how you can use numbers to create these same two equivalent fractions.



- 2. Use numbers to find 5 more fractions equivalent to 3/4.
- **3.** Use graph paper to find 2 fractions equivalent to 20/30. Explain what you did with your graph paper pictures. Describe how you can use numbers to create these same two equivalent fractions.

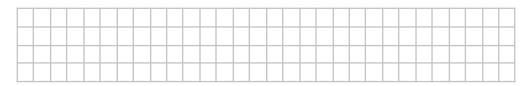


4. Find 5 fractions equivalent to 200/300 but with numerators less than 200.

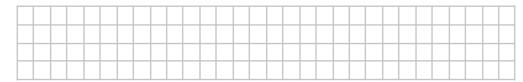
Student Sheet 52

1. Use graph paper to show that $\frac{2}{3} \times \frac{4}{5} = \frac{2 \times 4}{3 \times 5}$.

[Hint. Represent 1 with a rectangle having 3 rows and 5 columns; show 2/3 with rows and 4/5 with columns.]



2. Use graph paper to show that $\frac{3}{4} \times \frac{5}{7} = \frac{3 \times 5}{4 \times 7}$.



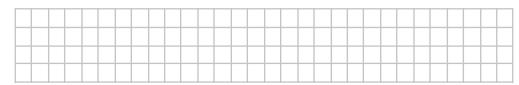
3. Use graph paper to show that $10 \times \frac{2}{3} = \frac{10 \times 2}{1 \times 3}$.

[Hint. Make 1 be a rectangle with 3 squares. Think of 10 copies of 2 thirds—that's 20 thirds.]



4. Use graph paper to show that $\frac{2}{3} \times 15 = \frac{2 \times 15}{3 \times 1}$.

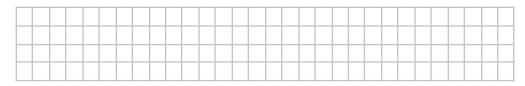
[Hint. Make 1 be one square. Think of this problem as 2/3 of 15.]



Student Sheet 53

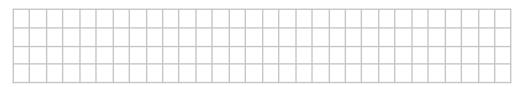
1. Use graph paper to show that $4 \div \frac{2}{3} = 4 \times \frac{3}{2}$.

[Hint: Make 1 contain a horizontal strip of 3 squares. Think about 4 as 1 + 1 + 1 + 1, and think about how many times $\frac{2}{3}$ goes into 4.]



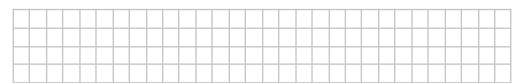
2. Use graph paper to show that $4 \div \frac{3}{5} = 4 \times \frac{5}{3}$.

[Hint: Make 1 contain a horizontal strip of 5 squares. Think about 4 as 1 + 1 + 1 + 1, and think about how many times $\frac{3}{5}$ goes into 4.]



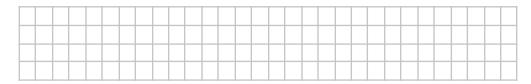
3. Use graph paper to show that $\frac{3}{4} \div \frac{2}{3} = \frac{3}{4} \times \frac{3}{2}$.

[Hint. Make 1 contain a horizontal strip of 12 squares. Think about how many times $\frac{2}{3}$ goes into $\frac{3}{4}$.]



4. Use graph paper to show that $\frac{2}{3} \div 4 = \frac{2}{3} \times \frac{1}{4}$.

[Hint. Make 1 contain a horizontal strip of 12 squares. Think of dividing 2/3 into 4 equal parts.]

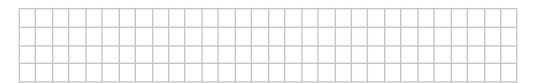


Name	Date

Student Sheet 53 (Continued)

5. Use graph paper to show that $4 \div 3 = 4 \times \frac{1}{3}$.

[Hint. Make 1 contain a horizontal strip of 3 squares. Think of dividing 4 among 3 people.]



Student Sheet Answers

Although answers are provided for CBA student sheets, when assessing students' work on the sheets it is essential to determine the CBA levels of reasoning students use.

STUDENT SHEET 1

- 1. 4 cookies
- 2. Solution 1: 1 whole square + 1/4 of another (but not named as fractions) Solution 2: 1/4 of each square

STUDENT SHEET 2

- 1. shade any 1 square; 1/6
- 2. shade any 1 column
- 3. shade any 2 columns
- **4A.** 1/5
- **4B.** 1/8
- **4C.** Can't tell. It's not 1/2 because the 2 pieces are not equal.

STUDENT SHEET 3

- 1. shade any 2 columns
- 2. shade any 4 columns
- 3. shade any 3 columns
- **4.** shade any set of 5 sectors

STUDENT SHEET 4

- 1. numerous: e.g., any 2 columns; any set of 6 squares; subdivision by diagonal
- 2. numerous: e.g., any row; any set of 4 squares
- 3. numerous: e.g., any column; any set of 6 squares
- **4.** numerous: e.g., any set of 2 squares

STUDENT SHEET 5

1/2 (show by iterating trapezoid 2 times); 1/3 (show by iterating rhombus 3 times)

STUDENT SHEET 6

1/6 (show by iterating triangle 6 times); 1/4 (show by iterating trapezoid 4 times)

1/2 (show by iterating hexagon 2 times); 1/6 (show by iterating rhombus 6 times)

STUDENT SHEET 8

1/12 (show by iterating triangle 12 times); 1/3 (show by iterating triangle 3 times); 1/2 (show by iterating triangle 2 times)

STUDENT SHEET 9

- 1. numerous: e.g., any 2 rows; any set of 8 squares
- 2. numerous: e.g., any 3 columns; any set of 9 squares
- 3. numerous: e.g., any 4 rows; any set of 16 squares
- 4. numerous: e.g., any set of 6 squares

STUDENT SHEET 10

Answers are shown by iterating the pattern block named below (in parentheses), first for the whole, then for the part.

- 1. 1/2 (trapezoid); 4/6 (triangle), also 2/3 (rhombus); 2/6 (triangle), also 1/3 (rhombus)
- 2. 8/12 (triangle), also 4/6 (rhombus) [also 2/3]; 8/12 (triangle), also 4/6 (rhombus) [also 2/3]; 6/12 (triangle), also 3/6 (rhombus) [also 1/2]
- **3.** 3/10 (triangle); 4/10 (triangle), 2/5 (rhombus)

STUDENT SHEET 11

- 1. 4/12 (triangle), also 2/6 (rhombus) [also 1/3]; 6/10 (triangle), 3/5 (rhombus)
- **2.** 6/9 (triangle)

STUDENT SHEET 12

- 1. 3/6 (square), also 1/2
- **2.** 4/8 (square), also 1/2
- 3. 6/8 (square), also 3/4
- **4.** 1/8 (square)

STUDENT SHEET 13

Answers are shown by iterating the pattern block named (in parentheses) below, first for the whole, then for the part.

- **1.** 1/4 (rhombus)
- 2. 1/2 (shaded parallelogram); 4/8 (triangle)

- **3.** 3/8 (triangle)
- **4.** 5/8 (triangle)

Show by iterating pattern blocks:

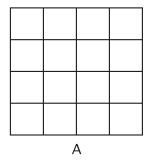
The triangle is NOT one-third of the whole.

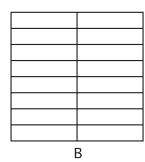
The trapezoid is NOT one-third of the whole.

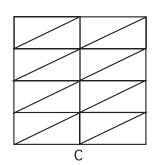
The rhombus IS one-third of the whole.

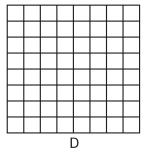
STUDENT SHEET 15

- 1. Use grid A to show that 12/16 of the large square is shaded (also 6/8 and 3/4).
- **2.** Use grid A to show that 7/16 of the large square is shaded.
- **3.** Use grid A to show that 9/16 of the large square is shaded.
- **4.** Use grid B to show that 8/16 of the large square is shaded (also 4/8, 2/4, 1/2).
- **5.** Use grid C to show that 6/16 of the large square is shaded (also 3/8).
- **6.** Use grid D to show that 12/64 of the large square is shaded (also 6/32, 3/16).

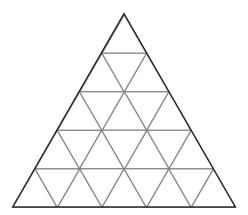








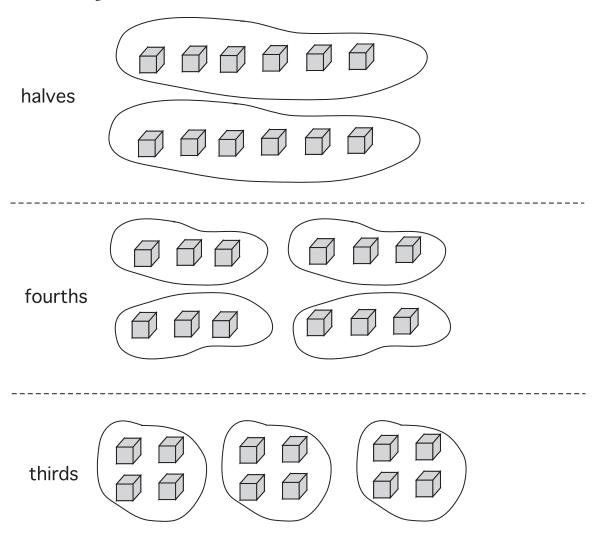
7. Use the grid below to show that 15/25 of the large triangle is shaded.



Students explain how they found answers, or prove them, by iterating the first specified rod in the answers below. Answers can be multiple rods, or one rod. Not all combinations are given.

- 1. 1 red; 2 reds or 1 purple; 3 reds or 1 dark green
- 2. 1 red; 2 reds or 1 purple; 3 reds or 1 dark green
- **3.** 1 white; 2 whites or 1 red; 5 whites or 1 yellow
- **4.** 1 white; 3 whites or 1 light green; 4 whites or 1 purple
- **5.** 1 white; 4 whites or 1 purple; 7 whites or 1 black

For Student Sheets 17 through 19, students find answers by putting cubes in groups (or circling cubes on the student sheet).



- **1.** 6 cubes
- **2.** 4 cubes
- **3.** 3 cubes
- **4.** 12 cubes
- **5.** 8 cubes

STUDENT SHEET 18

- **1.** 6 cubes
- **2.** 3 cubes
- **3.** 4 cubes

STUDENT SHEET 19

- **1.** 9 cubes
- **2.** 10 cubes
- **3.** 20 cubes
- **4.** 18 cubes
- **5.** 10 cubes

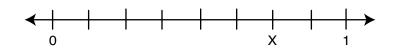
STUDENT SHEET 20

- 1. shade 4 squares
- 2. shade 8 squares
- 3. shade 10 squares
- **4.** shade 9 squares

STUDENT SHEET 21

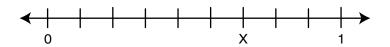
Subdivide the number line into equal segments.

- **1.** 3/8
- **2.** 5/8
- 3.



4. 4/6 or 2/3

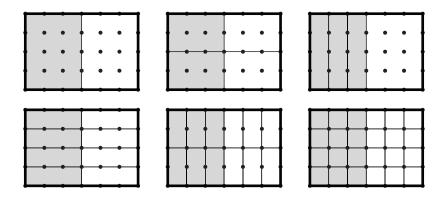
5.



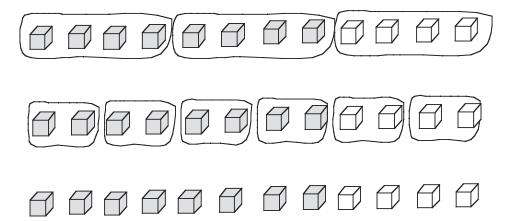
6. 1/2 [note where 2 is, find 1, then find X]

STUDENT SHEET 22

1.

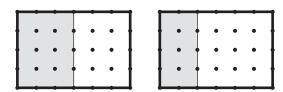


2. 2/3, then 3/4, then 8/12.



STUDENT SHEET 23

1. Place one shaded part on the other to compare directly, or count columns or squares, to show that 1/2 > 1/3.



2. Use drawings similar to Problem 1.

For 3/4 shade 3 rows. For 2/3 shade 4 columns (2 sets of 2) in another color. Cut up pieces, or count squares, to show that 2/3 < 3/4.

3. Use drawings similar to Problem 1.

For 3/6 shade 3 columns. For 2/4 shade 2 rows in another color. Cut up pieces, or count squares, to show that 3/6 = 2/4.

4. Use drawings similar to Problem 1.

For 5/6 shade 5 columns. For 3/4 shade 3 rows in another color. Cut up pieces, or count squares, to show that 5/6 > 3/4.

5. Use drawings similar to Problem 1.

For 5/6 shade 5 columns. For 11/12 shade $11\ 1$ by 2 rectangles in another color. Cut up pieces, or count squares, to show that 5/6 < 11/12.

STUDENT SHEET 24

- **1.** 16 squares, 8 squares, 8/16; 1/2 (use 2 by 4 rectangles); 4/8 (use 2 by 1 rectangles); 2/4 (use rows)
- **2.** 16 squares, 8 squares, 8/16; 1/2 (use 2 by 4 rectangles); 4/8 (use 2 by 1 rectangles); 2/4 (use rows)
- **3.** 16 squares, 12 squares, 12/16; 3/4 (use 2 by 2 squares or rows); 6/8 (use 2 by 1 rectangles)

STUDENT SHEET 25

- 1. Iterate red rod 5 times to show 1 (orange).
- **2.** Iterate light green rod 3 times to show 1 (blue).
- **3.** Iterate light green rod 2 times to show 2/3 (dark green), then iterate light green rod 3 times to show 1 (blue).
- **4.** Iterate red rod 2 times to show 2/5 (purple), then iterate red rod 5 times to show 1 (orange).

Challenge Problem

5. Iterate red rod 3 times to show 1 (dark green). Iterate white rod 6 times to show 1 (dark green). Iterate white rod 5 times to show 5/6 (yellow).

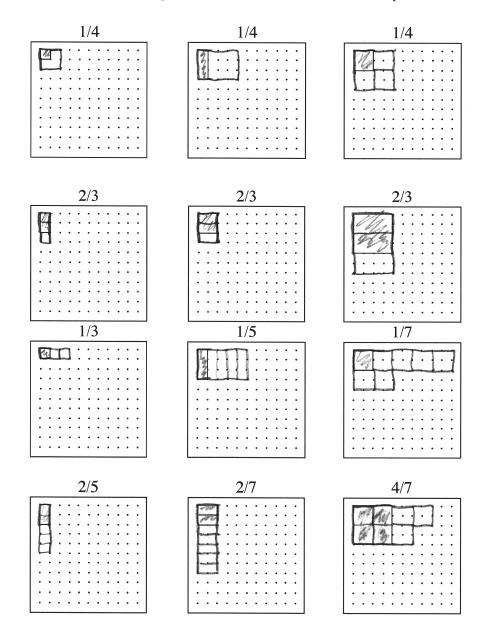
STUDENT SHEET 26

- 1. Iterate red rod 4 times to show 1 is brown.
- **2.** Iterate white rod 5 times to show 1 is yellow.
- **3.** Iterate light green rod 3 times to show 1 is blue.

- **4.** Iterate red rod 3 times to show 3/5 (dark green). Iterate red rod 5 times to show 1 (orange). [Proof: Iterate red rod to show that dark green is 3/5 of orange.]
- **5.** Iterate white rod 4 times to show 4/5 (purple). Iterate white rod 5 times to show 1 is yellow. [Proof: Iterate white rod to show that purple is 4/5 of yellow.]
- **6.** Iterate white rod 5 times to show 5/8 (yellow). Iterate white rod 8 times to show 1 is brown. [Proof: Iterate white rod to show that yellow is 5/8 of brown.]

See picture below.

For each problem, a fractional part of the whole is shown. Find the whole. Explain.

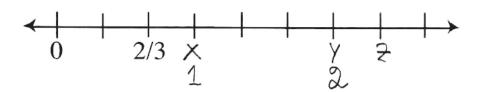


- **1.** 15 cubes
- **2.** 8 cubes
- **3.** 9 cubes
- **4.** 14 cubes
- 5. 16 cookies

Challenge Problems

6. 1/8 of the whole bag iterated 5 times makes 40. So 1/8 of the whole bag is $40 \div 5 = 8$. Thus the whole bag is 8/8, which in this case, is 64 jelly beans. Naomi ate 1/4 of 64 = 16 jelly beans. Mario and Naomi ate a total of 40 + 16 = 56 jelly beans. So there are 8 jelly beans left, which is 1/8 of the whole bag.

7.



The rectangle is 1. Show 2.	The rectangle is 1. Show 1 1/2.						
The rectangle is 1. Show 2.	The rectangle is 1. Show 1 1/2.						
The rectangle is 1. Show 3/2.	The rectangle is 1. Show 3/2.						
The rectangle is 1. Show 7/4.	The rectangle is 1. Show 7/4.						

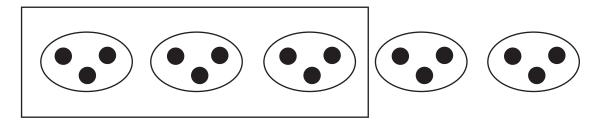
extangle is 1. Show 13/6. The rectangle is 1. Show 13/6.	rectangle is 1. Show 5/4.	The rectangle is 1. Show 5/4.				
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ctangle is 1. Show 13/6. The rectangle is 1. Show 13/6.	* * * * * * *	* * * * * * * * * * * * * * * * * * *				
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ctangle is 1. Show 13/6. The rectangle is 1. Show 13/6.	* * * * * * * * * * * * * *					
ctangle is 1. Show 13/6. The rectangle is 1. Show 13/6.	* * * * * * * * * * * * * *					
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	ctangle is 1. Show 13/6.	The rectangle is 1. Show 13/6.				
	ctangle is 1. Show 13/6.	The rectangle is 1. Show 13/6.				
	ctangle is 1. Show 13/6.	The rectangle is 1. Show 13/6.				
	etangle is 1. Show 13/6.	The rectangle is 1. Show 13/6.				
	etangle is 1. Show 13/6.	The rectangle is 1. Show 13/6.				
	ctangle is 1. Show 13/6.	The rectangle is 1. Show 13/6.				
	ctangle is 1. Show 13/6.	The rectangle is 1. Show 13/6.				
	extangle is 1. Show 13/6.	The rectangle is 1. Show 13/6.				
	etangle is 1. Show 13/6.	The rectangle is 1. Show 13/6.				

- **1.** 1/2 of 12 cubes is 6 cubes, so 3/2 is 18 cubes.
- **2.** 1/2 of 12 cubes is 6 cubes, so 5/2 is 30 cubes.
- **3.** 1/4 of 12 cubes is 3 cubes, so 7/4 is 21 cubes.

STUDENT SHEET 31

- 1. 1 cube, 2 cubes, 4 cubes, 8 cubes; divide the number of cubes by 3; $36 \div 3 = 12$ cubes
- 2. 2 cubes, 4 cubes, 8 cubes, 16 cubes; divide the number of cubes by 3, then multiply by 2; $36 \div 3 = 12$, $12 \times 2 = 24$ cubes
- 3. 1 cube, 2 cubes, 4 cubes; divide the number of cubes by 5; $30 \div 5 = 6$ cubes
- **4.** 3 cubes, 9 cubes, 12 cubes; divide the number of cubes by 5, then multiply by 3; $30 \div 5 = 6$, $6 \times 3 = 18$ cubes
- **5.** divide 40 by 8 (= 5), then multiply by 3 (= 15)

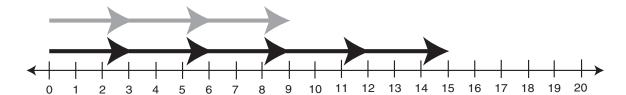
- **a.** Link 15 cubes together; this is 1. Separate 1 into 5 equal groups to show fifths. Select 3 groups (= 9 cubes).
- **b.** Draw a 15 by 1 rectangle; this is 1. Separate 1 into 5 equal groups to show fifths. Select 3 groups (= 9 squares).
- **c.** Draw 15 dots as 5 groups of 3; select 3 groups; 9 dots.



d. Make a row of 15 white rods; iterate light green rod 5 times to show that the light green is 1/5 of 15; 3 light greens is 9 whites; so 3/5 of 15 equals 9.

White	White	White	White	White	White	White	White	White	White	White	White	White	White	White
Light Green		Li	ght Gre	en										

e. Draw 5 arrows of length 3 to 15. Each arrow is 1/5; 3 of the arrows end at 9.

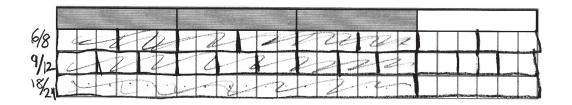


See picture below.

1. Draw 3 fractions equivalent to 1/2. Label each fraction you make.



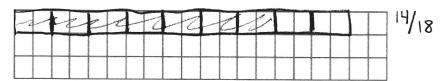
2. Draw 3 fractions equivalent to 3/4. Label each fraction you make.



3. Use pictures to find a fraction that is equivalent to 3/5 and has a denominator of 20.

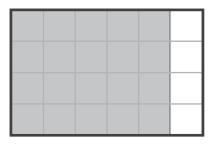


4. Use pictures to find a fraction that is equivalent to 7/9 and has a denominator of 18.

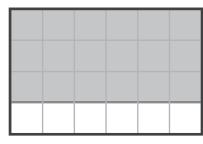


See pictures below.

1. 5/6 > 3/4

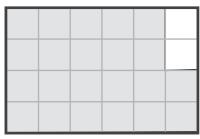


$$\frac{5}{6} = \frac{20}{24}$$

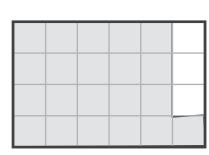


$$^{3}/_{4} = ^{18}/_{24}$$

2. 11/12 > 7/8

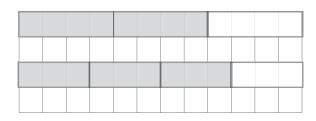


$$^{11}/_{12} = ^{22}/_{24}$$



$$^{7}/_{8} = ^{21}/_{24}$$

3. 3/4 > 2/3



$$^{2}/_{3} = ^{8}/_{12}$$

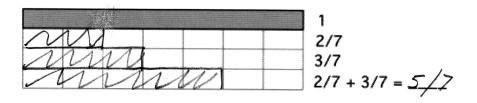
 $^{3}/_{4} = ^{9}/_{12}$

STUDENT SHEET 35

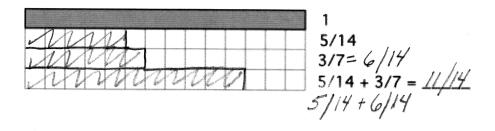
- 1. No, the parts are not equal, the unit 1 changes from rod 1 to rod 2.
- 2. No, the parts are not equal, so you can't tell from the picture what the fraction is.
- **3.** Yes, this is correct because she got a common denominator.

See picture below.

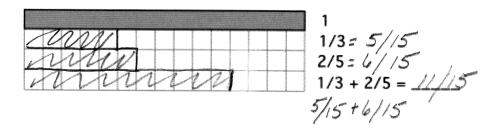
1. Use the drawing below to find $\frac{2}{7} + \frac{3}{7}$.



2. Use the drawing below to find $\frac{5}{14} + \frac{3}{7}$.

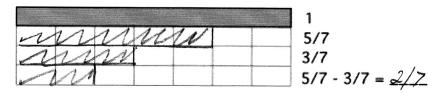


3. Use the drawing below to find $\frac{1}{3} + \frac{2}{5}$.

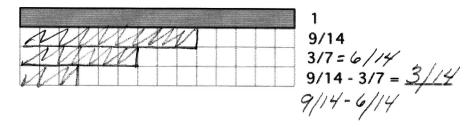


See picture below.

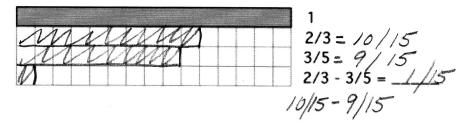
1. Use the drawing below to find $\frac{5}{7} - \frac{3}{7}$.



2. Use the drawing below to find $\frac{9}{14} - \frac{3}{7}$.



3. Use the drawing below to find $\frac{2}{3} - \frac{3}{5}$.



STUDENT SHEET 38

1. The rhombus is 1/3 because 3 rhombuses cover the hexagon. The trapezoid is 1/2 because 2 trapezoids cover the hexagon.

Using triangles, you can see that 1/2 = 3/6 and 1/3 = 2/6. So 1/2 + 1/3 = 5/6 (5 triangles).

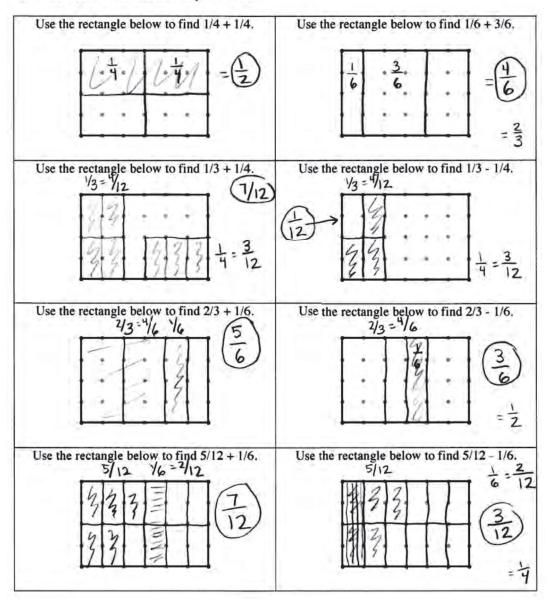
Using triangles, you can see that 1/2 = 3/6 and 1/3 = 2/6. So 1/2 - 1/3 = 1/6 (1 triangle).

Cuisenaire Rods

2. Similar method with Cuisenaire rods.

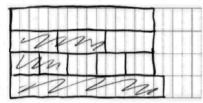
See picture below.

For each problem, find a common denominator to help you add or subtract the fractions. What the lowest common denominator you can use?



See picture below.

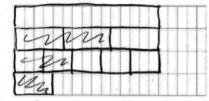
Problem. $\frac{2}{3} + \frac{2}{5}$



Draw the whole, or 1.

Draw the first fraction. $\frac{2}{3} = \frac{10}{15}$ Draw the second fraction. $\frac{2}{5} = \frac{6}{15}$ Draw the sum or difference. $\frac{16}{5} = \frac{10}{15}$

Problem. $\frac{2}{3} - \frac{2}{5}$



Draw the whole, or 1.

Draw the first fraction. $\frac{2}{3} = \frac{10}{15}$ Draw the second fraction. $\frac{2}{5} = \frac{6}{15}$ Draw the sum or difference. $\frac{4}{15}$

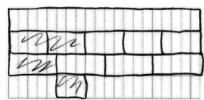
Problem. $\frac{1}{4} + \frac{2}{5}$



Draw the whole, or 1.

Draw the first fraction. |4| = 5/20Draw the second fraction. 2/5 = 8/20Draw the sum or difference. |3/20|

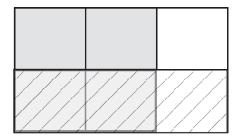
Problem. $\frac{2}{5} - \frac{1}{4}$



Draw the whole, or 1.

Draw the first fraction. $^{2}/_{5} = ^{8}/_{20}$ Draw the second fraction. $^{1}/_{4} = ^{5}/_{20}$ Draw the sum or difference. $^{3}/_{20}$

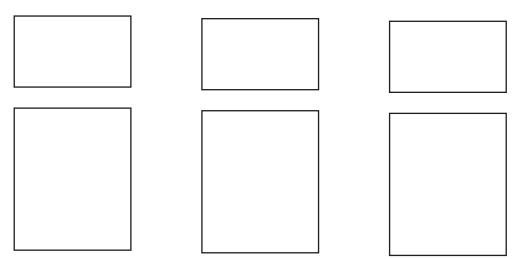
- **1.** Draw 12 dots. Circle 3 groups of 4. Select 2 groups = 8 dots.
- **2.** Draw 21 dots. Circle 7 groups of 3. Select 3 groups = 9 dots.
- 3. Double shaded part is 1/2 of 2/3 = 2/6.



- 4. Drawing similar to 3. 12/21
- **5.** 13
- **6.** 17 1/2

STUDENT SHEET 42

- **1.** Draw 3 rectangles, each with 5 squares. Each square is 1/5. So there are 15 fifths in 3.
- **2.** Draw 5 1/2 rectangles, each rectangle having 4 squares. Each square is 1/4. So there are 22 fourths in 5 1/2.
- **3.** Draw 6 rectangles, each with 3 squares. Each square is 1/3. Count groups of 2 squares. So there are 9 groups.
- **4.** Think of 4 1/2 as 3 3/2, which when divided into 3 equal groups gives 1 1/2 per group.



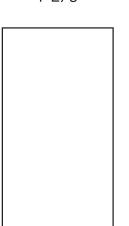
5. Think of $6\frac{2}{3}$ as $4\frac{8}{3}$. But 4 8/3 can be partitioned into 4 groups of 1 2/3.

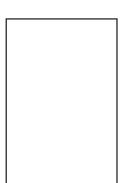
1 2/3

1 2/3

1 2/3

1 2/3

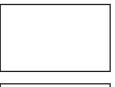


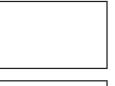






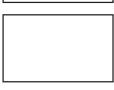














See picture below.

1.

Use the rectangles to find 5 x 1/2. [One rectangle should represent 1 whole or 1.]



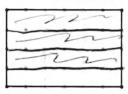


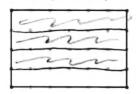




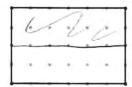


Use the rectangles to find 2 x 3/4. [One rectangle should represent 1 whole or 1.]

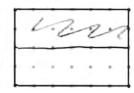




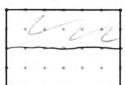
Use the rectangles to find 1/2 of 3. [One rectangle should represent 1 whole or 1.]

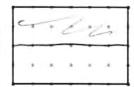


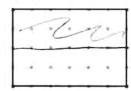




Use the rectangles to find 3 x 1/2. [One rectangle should represent 1 whole or 1.]







$$\frac{3}{2} = |\frac{1}{2}|$$

2. Use Cuisenaire rods to find $1/4 \times 2$. Explain how you found your answer.

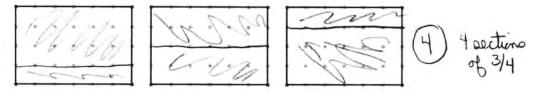
Ora	nge	Ora	nge
Yellow	Yellow	Yellow	Yellow

Make the orange rod 1. Because 4 copies of the yellow rod is 2, the yellow rod is 1/4 of 2. But 2 copies of the yellow rod is 1, so the yellow rod is 1/2.

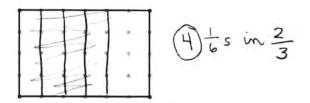
See picture below.

1.

Use the rectangles to find $3 \div 3/4$, [One rectangle should represent 1 whole or 1.]



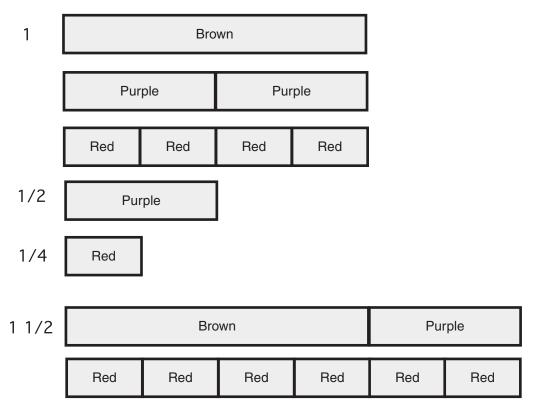
Use the rectangle to find $2/3 \div 1/6$. [One rectangle should represent 1 whole or 1.]



Use the rectangles to find $1.1/2 \div 1/4$. [One rectangle should represent 1 whole or 1.]

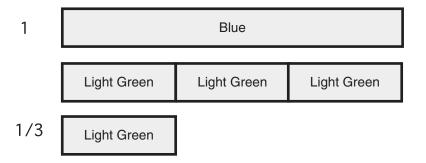


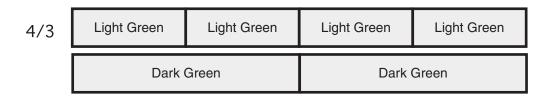
2. Use Cuisenaire rods to find $1 \frac{1}{2} \div \frac{1}{4}$. Explain how you found your answer.



The bottom part of the picture shows that there are 6 copies of 1/4 in 1 1/2.

3. Use Cuisenaire rods to find $4/3 \div 2$. Explain how you found your answer.



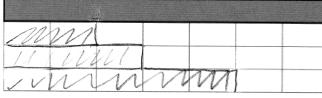


The bottom part of the picture shows that when 4/3 is divided into 2 equal parts, there are 2/3 in one of those parts.

See picture below.

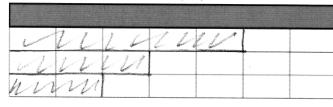
Use drawings to solve the problems. In each problem, the two numbers added or subtracted have the same denominator. Describe a procedure for adding or subtracting two numbers with the same denominator.

1. Use the drawing below to find $\frac{2}{7} + \frac{3}{7}$.



1

2. Use the drawing below to find $\frac{5}{7} - \frac{3}{7}$.



1
5/7
3/7
5/7 - 3/7 =
$$\frac{2}{7}$$

3. Use graph paper to find $\frac{1}{z} + \frac{2}{z}$.

	-		_	5 5	5				
0.0000000000000000000000000000000000000		especialistical Specialistical Companyation	STR 1	and the second	inate not	1			
AND DESCRIPTIONS	W					1/	5		
	W	11				2/	5		
200000000000000000000000000000000000000	le marine de	11	W		***************************************	1%	5 +	2/5=	3/5
									•

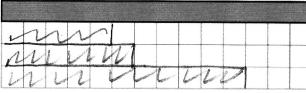
4. Use graph paper to find



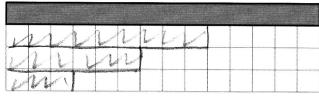
See picture below.

Use <u>drawings</u> to solve the problems. In each problem, the denominator of one of the numbers is a multiple of the denominator of the other number. Describe a procedure for adding or subtracting two numbers in which the denominator of one of the numbers is a multiple of the denominator of the other number.

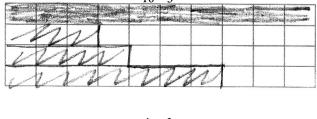
1. Use the drawing below to find $\frac{5}{14} + \frac{3}{7}$.

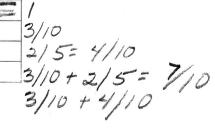


2. Use the drawing below to find $\frac{9}{14} - \frac{3}{7}$.

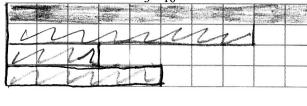


3. Use graph paper to find $\frac{3}{10} + \frac{2}{5}$.





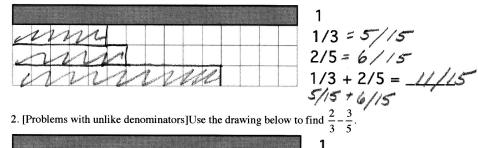
4. Use graph paper to find $\frac{4}{5} - \frac{3}{10}$.

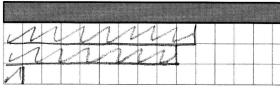


See picture below.

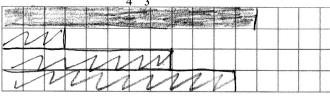
Use drawings to solve the problems. Describe a procedure for adding or subtracting two numbers that will work for any denominators.

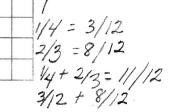
1. [Problems with unlike denominators] Use the drawing below to find $\frac{1}{3} + \frac{2}{5}$.





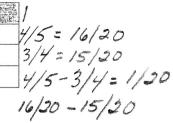
3. Use graph paper to find $\frac{1}{4} + \frac{2}{3}$. Decide how many squares you should put in 1.





4. Use graph paper to find . Decide how many squares you should put in 1.





1.
$$\frac{2}{3} \times \frac{6}{1} = \frac{12}{3}$$
 There are 12 shaded parts of size 1/3.

2.
$$\frac{3}{7} \times \frac{14}{1} = \frac{42}{7}$$
 There are 42 shaded parts of size 1/7.

3.
$$\frac{1}{2} \times \frac{2}{3} = \frac{2}{6}$$
 2 pieces of size 1/6 are double shaded.

4.
$$\frac{2}{3} \times \frac{6}{7} = \frac{12}{21}$$
 12 pieces of size 1/21 are double shaded.

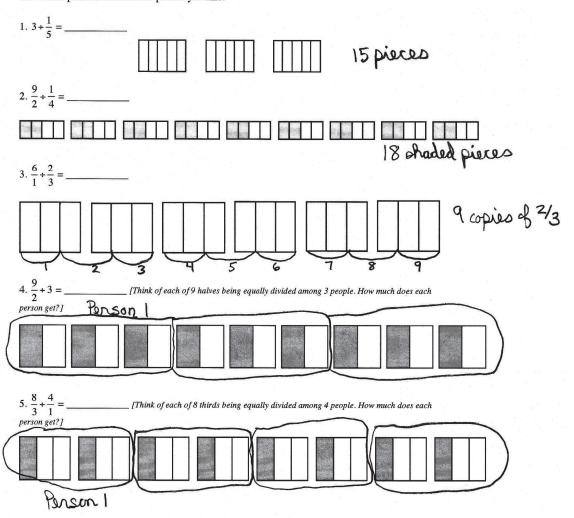
5.
$$\frac{3}{1} \times \frac{7}{3} = \frac{21}{3}$$
 There are 21 shaded parts of size 1/3.

6.
$$\frac{4}{1} \times \frac{7}{2} = \frac{28}{2}$$
 There are 28 shaded parts of size 1/2.

Pattern: To multiply two fractions, multiply the numerators (to find the numerator of the product), and multiply the denominators (to find the denominator of the product).

See picture below.

Use <u>drawings</u> to solve the problems. Find a pattern for how to manipulate the numerators and denominators of the dividend and divisor to find the numerator and denominator of the quotient. Write all numbers as fractions, not whole numbers or mixed numerals. Do more problems if you don't see a pattern. Describe the pattern you find.



- 1. $3 \div \frac{1}{5} = 15$ There are 15 pieces of size 1/5 in 3.
- 2. $\frac{9}{2} \div \frac{1}{4} = 18$ There are 18 pieces of size 1/4 in 9/2.
- 3. $\frac{6}{1} \div \frac{2}{3} = 9$ There are 9 groups of 2 pieces of size 1/3 in 6.
- 4. $\frac{9}{2} \div 3 = \frac{3}{2}$ If 9/2 is divided equally among 3 people, each person gets 3/2.

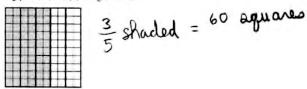
5.
$$\frac{8}{3} \div \frac{4}{1} = \frac{2}{3}$$
 If 8/3 is divided equally among 4 people, each person gets 2/3.

Pattern: To divide two fractions, invert the second fraction (exchange numerator and denominator) and multiply.

STUDENT SHEET 50

See picture below.

1. Explain how to find 3/5 of 100 (a) using pictures, and (b) using numbers.



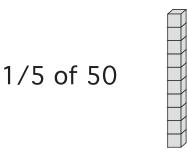
2. Find 3/4 of 1000. Describe how you found your answer.

3. Find 2/3 of 369. Describe how you found your answer.

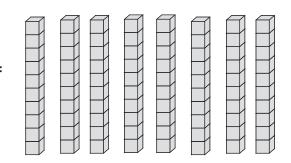
4. Use numbers and pictures to find 6/5 of 25. Describe how you found your answer

5 Use numbers and pictures to find 8/5 of 50. Describe how you found your answer.

6. Find 8/5 of 100. Describe how you found your answer.



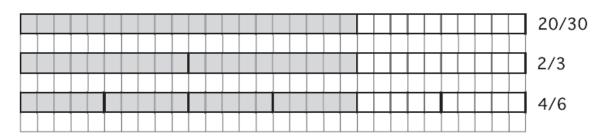
8/5 of 50 = 8 copies of 1/5 of 50



STUDENT SHEET 51

- 1. (a) Make a 12 by 1 rectangle, divide it into 4 equal sections, then shade 3 of the sections; there are 12 total squares and 9 shaded. So 3/4 = 9/12. In numbers, multiply the numerator and denominator of 3/4 by 3. (b) Make an 8 by 1 rectangle, divide it into 4 equal sections, then shade 3 of the sections; this is 3/4. Now count the squares in the sections; there are 8 total squares and 6 shaded. So 3/4 = 6/8. In numbers multiply the numerator and denominator of 3/4 by 2.
- **2.** Multiply the numerator and denominator of 3/4 by 4 to get 12/16, by 5 to get 15/20, by 6 to get 18/24, by 10 to get 30/40, by 25 to get 75/100.

3.

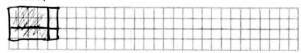


4. Divide the numerator and denominator of 200/300 by various numbers that evenly divide both 200 and 300 (such as 100, 10, 5, 20).

See picture below.

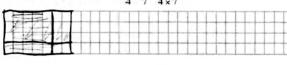
1. Use graph paper to show that $\frac{2}{3} \times \frac{4}{5} = \frac{2 \times 4}{3 \times 5}$

[Hint. Represent 1 with a rectangle having 3 rows and 5 columns: show 2/3 with rows and 4/5 with



2x4 squares double shaded 3x5 total squares

2. Use graph paper to show that $\frac{3}{4} \times \frac{5}{7} = \frac{3 \times 5}{4 \times 7}$



3×5 oquares double shaded 4×7 total oquares

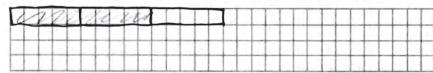
3. Use graph paper to show that $10 \times \frac{2}{3} = \frac{10 \times 2}{1 \times 3}$

[Hint. Make 1 be a rectangle with 3 squares. Think of 10 copies of 2 thirds-that's 20 thirds.]

2 405	2	325	3	12	-6	Z	7	3	each ophane = 3
								Till	0
	1								20 squares shad
									(20 = 10×2)

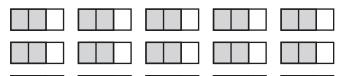
4. Use graph paper to show that $\frac{2}{3} \times 15 = \frac{2 \times 15}{3 \times 1}$.

[Hint. Make 1 be one square. Think of this problem as 2/3 of 15.]



10 oquares

[Note for 4. You can also recognize that $\frac{2}{3} \times 15 = 15 \times \frac{2}{3}$. Think of this as 15 groups of 2/3, which could be drawn as shown below. There are 30 pieces of size 1/3, or 10.]



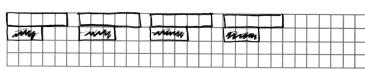
See picture below.

1. Use graph paper to show that $4 \div \frac{2}{3} = 4 \times \frac{3}{2}$. [Hint: Make 1 contain a horizontal strip of 3 squares. Think

about 4 as 1+1+1+1, and think about how many times $\frac{2}{3}$ goes into 4.]



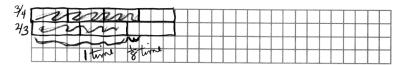
2. Use graph paper to show that $4 \div \frac{3}{5} = 4 \times \frac{5}{3}$. [Hint: Make 1 contain a horizontal strip of 5 squares. Think about 4 as 1+1+1+1, and think about how many times $\frac{3}{5}$ goes into 4.]



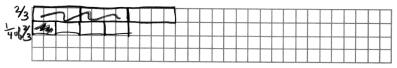
 $|\frac{3}{5}| = |\frac{2}{3}| = \frac{5}{3}$

3. Use graph paper to show that $\frac{3}{4} \div \frac{2}{3} = \frac{3}{4} \times \frac{3}{2} \cdot \frac{9}{8} = 1$

[Hint. Make 1 contain a horizontal strip of 12 squares. Think about how many times $\frac{2}{3}$ goes into $\frac{3}{4}$.]



4. Use graph paper to show that $\frac{2}{3} \div 4 = \frac{2}{3} \times \frac{1}{4}$. [Hint. Make 1 contain a horizontal strip of 12 squares. Think of dividing 2/3 into 4 equal parts.]



5. Use graph paper to show that $4 \div 3 = 4 \times \frac{1}{3}$.

[Hint. Make 1 contain a horizontal strip of 3 squares. Think of dividing 4 among 3 people.]



1 person gets 4 thirds or 4/3