Date _

CBA Volume: Student Sheet 1

For each problem, decide which cube building has more room inside, or if they have the same amount of room. Then find two ways to use cubes to check your answers, one way that uses counting, and the other way that does not use counting.

Problem 1



For each building, predict how many cubes it takes to make the building, then check your prediction using cubes. The buildings are completely filled with cubes inside.

Be sure to predict and check for one problem before moving on to the next problem.

| | Building | Decomposed Building | Prediction Check |
|------------|----------|---------------------|------------------|
| Building 1 | | | |
| Building 2 | | | |
| Building 3 | | | |
| Building 4 | | | |
| Building 5 | | | |

The outside of this cube building is completely covered with paint.



| How many of the cubes have 0 faces painted? |
|---|
| How many of the cubes have 1 face painted? |
| How many of the cubes have 2 faces painted? |
| How many of the cubes have 3 faces painted? |
| How many of the cubes have 4 faces painted? |
| How many of the cubes have 5 faces painted? |
| How many of the cubes have 6 faces painted? |
| How many cubes are there in the building, altogether? |

1. Draw the three views for this cube building. Check your views with cubes and at the website.



2. Make one cube building that has these three views.







Тор



Right Side

How many cubes does it take to completely fill each box? The bottom layer of each box is shown. The dimensions of the box are in cubes. *Predict*, then make the box with paper and fill it with cubes to check. Check your prediction for a box before going on to the next box.





Predict how many cubes it takes to fill the boxes made by these patterns. Check with cubes. Hint: What does the bottom layer of cubes look like for the box? How many layers are there in the box?



May be photocopied for classroom use. © 2012 by Michael Battista from *Cognition-Based Assessment and Teaching of Geometric Measurement: Building on Students' Reasoning*. Portsmouth, NH: Heinemann.

Predict how many cubes it takes to make a building, then check with cubes.

| How many cubes are needed to make this building? Prediction Check | |
|--|--|
| 2. How many cubes are needed to make this building? Prediction Check <i>Hint: Can your answer for problem 1 help you answer problem 2?</i> | |
| 3. How many cubes are needed to make this building? Prediction Check Hint: Can your answer for problem 2 help you answer problem 3? | |
| 4. How many cubes are needed to make this building? Prediction Check <i>Hint: Can your answer for problem 3 help you answer problem 4?</i> | |

Bethany was thinking about the procedure that students in her class used to find the number of cubes in rectangular cube buildings or boxes. They multiplied the length times the width times the height.

Bethany thought that this procedure must be wrong, even though it gave correct answers. Demonstrating with a 5-by-4-by-3 cube building, she counted 5 cubes along the length, 4 cubes along the width, and 3 cubes along the height, then pointed to the corner cube (gray) and said to her classmates, "The corner cube gets counted once when you find the length, once for the width, and once for the height. So this method should be wrong."



What would you say to Bethany to help her understand why the procedure of multiplying the length times the width times the height is correct, and that it does not double-count any cubes?

1. Collin has some packages that each contain two identical cubes. He wants to know how many of these **packages** it takes to completely fill the rectangular box below.

Predict an answer, then make the box and check your answer with cubes. Remember that it takes 2 cubes to make 1 package. (The pattern for making the box appears on the next page.)



2. Predict and check how many of each of these packages fit in Collin's box. The packages cannot be broken apart.

| Package B | Package C | Package D | Package E |
|-----------|-----------|-----------|-----------|
| | | | |

3. Are there different ways that these packages can fit in the box? If so, describe them.

CBA Volume: Student Sheet 9 (Continued)

Pattern (Net) for Collin's Box



NOTE: Be sure the squares in the pattern match the size of the physical cubes students use to check their answers.

Date _

CBA Volume: Student Sheet 10

1. Collin has some packages that each contain two identical cubes. He wants to know how many of these <u>packages</u> will fit in the rectangular box below, including the packages that are already in the box. <u>The packages cannot be broken apart</u>.



How many packages fit in the box?

Explain exactly how Collin must arrange the packages in the box.

2. How many cube-shaped packages made from 8 cubes will fit in the above box? The packages cannot be broken apart.



1. How is the volume of Building A related to the volume of Building B? Justify your answer.



2. Think of sliding the cards in a deck of cards over a bit, as pictured below. How does the volume of the original deck of cards compare to the volume of the slanted deck? Explain.



3. What is the volume of the non-right rectangular prism below?



4. Use the ideas from Problems 1–3 above to develop a formula for the volume of a non-right rectangular prism (that is, a prism in which not all the vertex angles on the lateral sides are right angles). Justify your answer.



May be photocopied for classroom use. © 2012 by Michael Battista from *Cognition-Based Assessment and Teaching of Geometric Measurement: Building on Students' Reasoning*. Portsmouth, NH: Heinemann.

CBA Volume: Student Sheet 11 (Continued)

5. What do you think the formula for the volume of a triangular prism is? Why? How does the formula change, if at all, if the triangular prism is slanted (that is, it is not a right triangular prism)? What is the volume of the non-right triangular prism below?



6. What do you think the formula for the volume of a cylinder is? Why?



- 7. Find the volume of a triangular prism that is 10 cm high and has a right triangular base with legs that measure 7 and 12 cm.
- **8.** Find the volume of a cylinder whose circular base has a radius of 5 inches, and has a height of 20 inches.

Date _

CBA Volume: Student Sheet 12

Pyramid A and Prism B have congruent rectangular bases and the same height.

Cut out the patterns (nets) for the shapes on the next pages, make the shapes, and open a flap on their bottoms. Use sand to determine how many pyramids of sand it takes to completely fill the prism.



Pyramid A



CBA Volume: Student Sheet 12 (Continued)



Pyramid Net/Pattern

CBA Volume: Student Sheet 12 (Continued)

Prism Net/Pattern

Date __

CBA Volume: Student Sheet 13

Shown below is a rectangular prism that measures 3-by-5-by-7 cm. Find its volume.



Shown below are three rectangular pyramids—A, B, and C—constructed inside the 3-by-5-by-7 cm rectangular prism. Each pyramid has a face of the prism as its base. Find the volume of each pyramid.



How do the volumes of the pyramids compare? How are the volumes of the pyramids related to the volume of the prism?